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Stochastic Line Search Using UUVs

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Abstract - ***Unmanned underwater vehicles (UUVs) are increasingly being used in a diverse range of applications. In one particular application, we analyze UUV operations for location, detection and classification of mines. The mission objective is to search the area of interest, using underwater imaging sensors such as side scan sonars, until either the first mine is located, or it is verified that none can be found. Communication constraints require that the vehicle be connected with physically for downloads. In such as scenario, the search area can be considered as a line, and prior probabilities of finding a mine on the line can be related to external considerations such as the bottom characteristics, etc. The optimization problem is the determination of a sequence of points on the line where the UUV should be configured to return for a data download, so as to minimize the expected mission time. Operational models are defined and analytical expressions and numerical results describe the optimal strategies for searching with several distributions and return point specifications.***

Keywords: UUV Operations, stochastic search, resource allocation.

1 Introduction

Unmanned Undersea Vehicles (UUVs) are on the threshold of playing key roles in the battlespace [1], port protection and undersea surveying applications. Because of their autonomous operation, such vehicles are particularly attractive for conducting detailed scans of sensitive areas. However, this requirement for autonomy places some stringent limitations on the capabilities of such vehicles. When operating autonomously, they have limited communication and energy capacities.

The energy constraint limits the range and processing power of the UUV. Typically, UUVs can operate for three to eight hours whilst collecting data. In this mode they typically travel at speeds of three to four knots. The upper limits of speed are typically five to six knots, although some vehicles can attain speeds as high as nine knots.

Communication capabilities of UUVs are constrained by their operating environment. Although underwater communication is possible using either radio or light or acoustic channels, for now acoustic communication is most widely used and the technology supporting it is the most mature. Thus, the bandwidth available when the UUV is submerged is limited by the speed of sound in water [3], which is adequate for sending and receiving status and navigation messages. However, when the UUV is engaged in scans using side scan or forward looking sonar or multibeam echosounders, the volume of data collected is far greater than can be efficiently transmitted by acoustic communication modes. In such situations, the UUV can be programmed to return to the “home” vessel from where it was launched, and the data can be downloaded by direct contact. When ‘home”, the UUV’s energy can be replenished by either recharging or replacing the battery pack. This mode of operation, which has the advantage of covertness, limits the throughput capacity of the UUV considerably. An alternate approach is to program the UUV to rise to the surface and transmit the data at higher capacity bandwidths. Although this can speed operations, such an operation will result in greater visibility, and as a result of the energy constraints, may not give an extended advantage over the former.

2 Problem Formulation

In this paper we consider some operational problems associated with the use of single or multiple UUVs. Specifically, we develop a model that analyzes the use of UUVs for exploratory operations, where the task is to find a single object located in an area of interest. When engaged in the search, the UUV uses a sensor such as side scan sonar or an echo sounder and scans a broad swath of the seabed as it moves at either a fixed altitude or depth. The actual path of the UUV is usually computed assuming a standard ”lawn-mower” approach to completely cover the area of interest. Thus, regardless of the actual geometry of the search area, the problem can be considered to be a one-dimensional search problem. Our initial analysis assumes that exactly one item is located in the area of interest and the cost and probability functions are non-decreasing functions of the distance traveled on the line from the origin. We assume the probability density function $f(x)$ and cumulative distribution function $F(x)$ for the location of the object. We also assume that m non-intelligent rovers can travel along the line collecting data used toward locating the object. Although some UUVs have embedded target recognition capabilities (ATR), the capability of on-board (autonomous) Computer Aided Detection (CAD) and Computer Aided Classification (CAC) is limited. Because of this limited on-board processing capability, the UUVs themselves cannot process the data, they just collect it; hence, they do not know if they seen the object or not. This determination is made by operators or computers that are located on the base platform, which may be a ship where the UUV is launched from. UUVs currently have navigation accuracy of the order of 0.05% of the distance traveled [4], and thus it is reasonable to assume that they are capable of autonomous traversal on a straight line.

Most UUVs can be programmed to travel within a range of speeds. The upper limit of this range is usually determined by the motor configurations, and the lower end of the range is limited by the sensor design configuration. Side scan sonars and multi-beam echo sounders are designed to perform at a nominal operating speed. We denote this as v_c, which is the velocity of the UUV when collecting data. The maximum velocity, v_r is the velocity of the UUV when traveling at maximum speed along the line to a new search start or back to the base station ($v_r \geq v_c$). When traveling at v_r, the UUV does not collect data. An equivalent return velocity can also be computed when the UUV does not physically travel back to deliver the data by considering the time required to surface, transmit and resume scanning operations. In this case the ratio v_r/v_c may be considerably higher.

After scanning either a portion of or the entire of the area of interest, the UUV is programmed to return to the base station. On the straight line, this is placed, without loss of generality, at x coordinate 0. At this base station, the data from the UUV is downloaded for further analysis by either human operator(s) or powerful computers, or both. We assume that the data processing itself does not result in a delay in the mission. Although the time to review the data is not negligible, it is possible for the subsequent segments of the mission to be initiated before the data review is completed, and based on the results of the review, the mission underway can be terminated or allowed to proceed as planned for additional data collection.

We measure the performance of the mission vis-a-vis the average time to locate the object. The analysis in the remainder of this paper assumes that the object is indeed present in the area of interest. If there is a probability associated with the presence of the object, the results will be conditioned by this factor.

Let $s_k, k = 1, \ldots m$ be the initial starting point of UUV k. For now, we assume this starting point is identically at the location of the base station, at coordinate 0. We are interested in searching a line by sequentially searching simply connected segments (intervals) on the line. The general decision problem requires determination of the intervals (the starting and ending points for each data collection foray) as well as the order in which the segments are traversed. (Without loss of generality, we assume that the domain of the search space is the segment [0,1] with probability density function $f(x)$ and cumulative distribution function $F(x)$ describing the random nature of the target location on that interval.)

Define the interval endpoints as b_k, $k = 0, 1, 2, \ldots n$ with $b_0 = 0$, $b_n = 1$, and $b_k \leq b_{k+1}$.

Define the interval probabilities, $\mathbf{p} = [p_1, p_2, \ldots p_n]$, as $p_k = F(b_k) - F(b_{k-1})$ in which $F(\cdot)$ is the cdf of the target location.

Next, define the n by n matrix $\mathbf{X}$ to describe the search order for the intervals. Specifically, the entries of $\mathbf{X}$ are (row i, column j)

$$x_{i,j} = \begin{cases} 0 & ; \quad \text{if interval } i \text{ is searched after interval } j \\ 1 & ; \quad \text{otherwise} \end{cases}$$

For example, for search sequence 1, 3, and 2, the matrix $\mathbf{X}$ is

$$\mathbf{X} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Define the search probability $\mathbf{q} = [q_1, q_2, \ldots q_n]$ by

$$\mathbf{q} = \mathbf{X}\mathbf{p} \tag{1}$$

Searching each interval entails a cost: traveling to one of the endpoints, searching the interval itself, and returning to home from the other endpoint. As our interest is total time, we use the notation $T(a, b)$ to describe this cost for an interval with endpoints a and b (we assume $a \leq b$)

$$T(a, b) = \frac{a + b}{v_r} + \frac{b - a}{v_c} \tag{2}$$

in which v_r and v_c are the *running* and *collecting* velocities of the rover, respectively.

With these definitions, the cost of searching the n intervals is

$$\text{cost} = \sum_{k=1}^{n} T(b_{k-1}, b_k)\, q_k, \tag{3}$$

and the optimization problem is:

$$\begin{array}{lll} \text{Minimize} & \sum_{k=1}^{n} T(b_{k-1}, b_k)\, q_k & \\ \text{s.t.} & & \\ & b_i \leq b_{i+1} & i = 1 \ldots m \\ & b_i \in [0, 1] & \forall\, i \end{array}$$

3 Analytical and Numerical Solutions

As mentioned above, the time to search a segment $[a, b]$ is:

$$T(a,b) = \frac{b+a}{v_r} + \frac{b-a}{v_c}. \quad (4)$$

Thus, the time required to scan the entire line segment $[0, d]$, with the requirement that the UUV returns exactly once to the base from an intermediate point b is:

$$T_1(0,d) = \frac{b}{v_r} + \frac{b}{v_c} + (1 - F(b))(T(b,d)), \quad (5)$$

where $F(b)$ is the cumulative probability, $\int_0^b f(x)dx$.

If the search sequence is assumed to be $[1, 2, \ldots, n]$, the general expression for m returns can be stated as:

$$T_m(0,d) = \frac{b_1}{v_r} + \frac{b_1}{v_c} + (1 - F(b_1))\left(T_{(m-1)}(b_1,d)\right). \quad (6)$$

In this case, the optimization problem is:

$$\begin{aligned} Minimize \quad & T_m(0,d) \\ \text{s.t.} \quad & \\ b_i \quad & \leq b_{i+1}, \quad i = 1 \ldots m \\ b_i \quad & \in [0,1] \,\forall\, i \end{aligned}$$

We now consider specific instances of this problem by specifying the probability distributions defining the location of the object on the line.

3.1 Example 1: Uniform distribution for object

In this case F$(x) = x/d$ for $0 \leq x \leq d$, so

$$\begin{aligned} T_0(0,d) - \; & T_1(0,d)(b) \\ & = \frac{b}{d}\left(\frac{d-b}{v_c} + \frac{b+d}{v_r}\right) - \frac{2b}{v_r} \\ & = \left(\frac{1}{dv_r} - \frac{1}{dv_c}\right) b^2 + \left(\frac{1}{v_c} - \frac{1}{v_r}\right) b \end{aligned}$$

which simplifies to

$$T_0(0,d) - T_1(0,d) = \left(\frac{v_c - v_r}{dv_r v_c}\right)(b-d)\, b$$

This quadratic, zero at both $b = 0$ and $b = d$ (as expected) takes on its largest value at $b = d/2$:

$$T_0(0,d) - T_1(0,d)|_{\max} = \left(\frac{v_r - v_c}{4v_r v_c}\right) d$$

In the case of a constant velocity UUV, $v_c = v_r$, the gain is trivially zero no matter what the value of b is.

In this case, the optimum length of the segment for the uniform case is independent of the relative speeds of collecting and running. If we parameterize the collecting speed as a fraction of the running speed, $v_c = \beta v_r = \beta v$, then

$$T_0(0,d) - T_1(0,d)|_{\max} = \left(\frac{1-\beta}{4\beta}\right)\frac{d}{v}$$

which confirms the intuition that small β implies large gain.

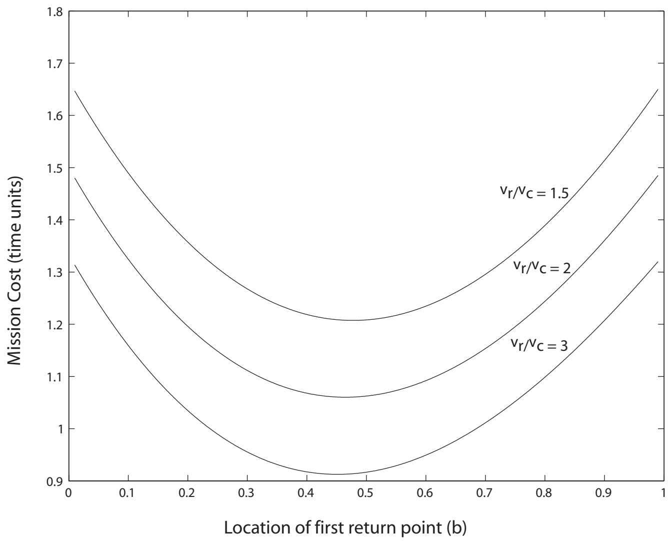

Figure 1: Cost as function of return point for triangular distribution

3.2 Example 2: Triangular distribution for object (with the peak at the origin)

The pdf in this case is:

$$f(x) = \begin{cases} 0 & ; \; x < 0 \\ \frac{2}{d^2}(d-x) & ; \; 0 < x < d \\ 0 & ; \; x > d \end{cases}$$

or

$$F(x) = \begin{cases} 0 & ; \; x < 0 \\ \frac{2}{d^2}\left(dx - \frac{x^2}{2}\right) & ; \; 0 < x < d \\ 1 & ; \; x > d \end{cases}$$

The Figure 1 shows a plot of the value of $T_1(0,d)$ as a function of the return point, (b). As can be seen in Figure 3.1, the cost of executing the mission varies parabolically with the location of the return point, and the return point is closer to the origin as the speed of return increases relative to the scan speed.

3.3 Example 3 - Piecewise constant (two section) distribution

In this case, the pdf and the cdf are respectively:

$$f(x) = \begin{cases} 0 & ; \; x < 0 \text{ or } x > d \\ p & ; \; 0 < x < a \\ q & ; \; a < x < d \end{cases}$$

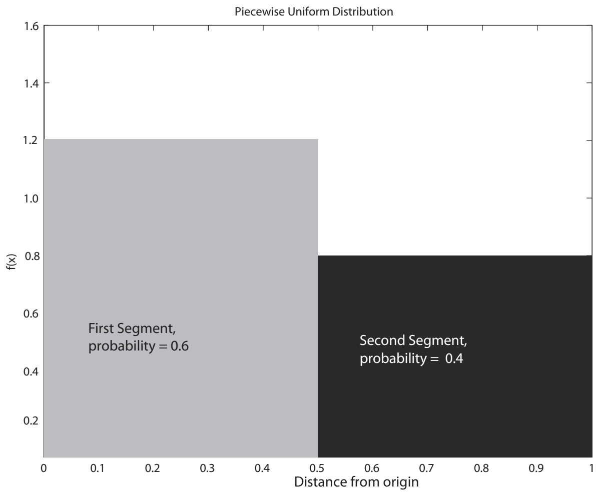

Figure 2: Return point vs p_1

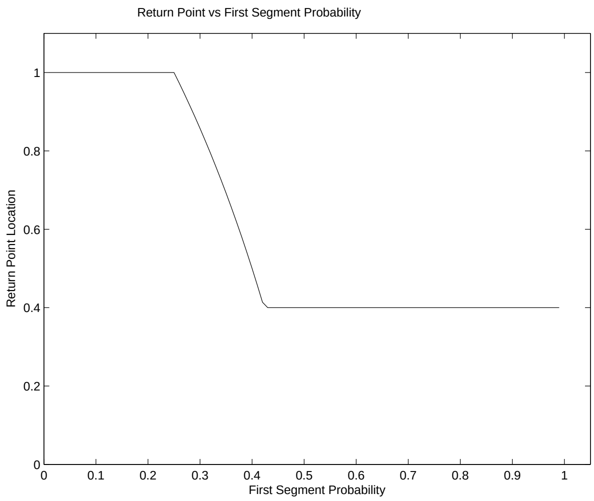

Figure 3: Return point vs p_1

and

$$F(x) = \begin{cases} 0 & ; \quad x < 0 \\ px & ; \quad 0 < x < a \\ pa + q(x-a) & ; \quad a < x < d \\ 1 & ; \quad x > d \end{cases}$$

where a, p, and q are such that $pa+q(d\text{-}a) = 1$. A typical two piece constant distribution is shown in Figure 2. Figure 3 shows the return point as a function of the probability weight of the first segment. As can be seen in this figure, as the total probability of finding the object in the first segment increases, the return point moves towards the origin. When the probability of finding the object in the first segment is low, the optimal solution for minimum mission time is one which takes the UUV to the end of the domain (located at a distance of one from the origin). As the first segment probability increases beyond a threshold, the return point moves progressively closer to the origin.

3.4 Example 4 - Two point returns

Consider the following probability distribution on the domain $[0, d]$:

$$f(x) = \frac{\pi}{2d} \cos \frac{\pi x}{2d} \quad ; \quad 0 < x < d \tag{7}$$

The cumulative distribution is:

$$F(x) = \sin \frac{\pi x}{2d} \quad ; \quad 0 < x < d \tag{8}$$

Using Equation (6), the solution for $m = 2$ (i.e. the mission is configured to bring back the UUV twice) with the above probability distribution is determined, numerically, to be $b = [.254, .606, .873]$. The parameters for this particular scenario were $v_r/v_c = 3$.

3.5 N point returns

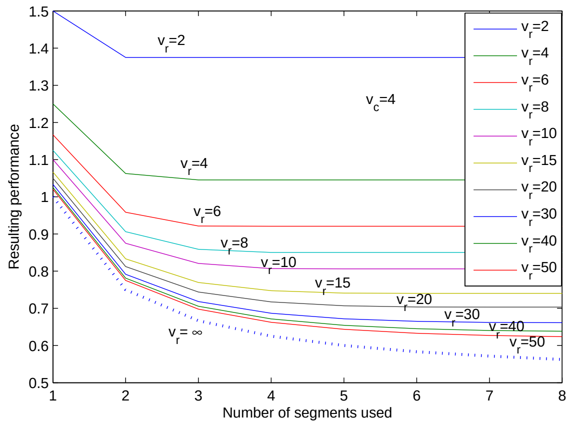

Figure 4: Mission time vs number of return points

To search the single interval $[a, b]$, $a < b$, the cost is the time spent running out to a, the time spent searching from a to b, and the time to return from b

$$\begin{aligned} T(a,b) &= \tfrac{a}{v_r} + \tfrac{b-a}{v_c} + \tfrac{b}{v_r} \\ &= b\left(\tfrac{v_r+v_c}{v_r v_c}\right) - a\left(\tfrac{v_r-v_c}{v_r v_c}\right) \\ &= \beta b - \alpha a \end{aligned}$$

Now, we assume that the search space [0,1] is subdivided into m intervals $[b_k, b_{k+1}]$ with $b_0 = 0$, $b_k \le b_{k+1}$, and $b_m = 1$, and that the search is sequential in the intervals. Since the search can terminate once the target is located, the average time for the search is just the sum of the products of the search time for each interval with the probability that interval must be searched

$$\text{average time } = \sum_{k=1}^{m} T(b_{k-1}, b_k)\,(1 - F(b_{k-1}))$$

in which $F(\cdot)$ is the cdf of the target location.

The Uniform Case

Let's assume a uniform pdf for the target on [0,1]; hence, the cdf is

$$F(x) = \begin{cases} 0 & ; \quad x \le 0 \\ x & ; \quad 0 < x < 1 \\ 1 & ; \quad 1 \le x \end{cases}$$

Substituting in, the cost is

$$\text{average time } = \sum_{k=1}^{m} \left(\beta b_k - \alpha b_{k-1}\right)\left(1 - b_{k-1}\right)$$

Let's optimize by taking a derivative with respect to one of the bs, say b_j. In doing so, two values of k in the sum are relevant, $k = j$ and $k = j + 1$. The derivative, after some simple algebra, is

$$\frac{\partial}{\partial b_j} = 2\alpha b_j - \beta\left(b_{j-1} + b_{k+1}\right) + (\beta - \alpha)$$

Setting this equal to zero and solving for b_j, we have

$$b_j = \frac{v_r + v_c}{v_r - v_c}\,\frac{b_{j-1} + b_{j+1}}{2} - \frac{v_c}{v_r - v_c}$$

In this form, it is possible to use the Gauss-Seidel algorithm for determining the solution in the following manner: hold the b_j, j odd, fixed and solve for the best b_j, j even; then hold the even terms fixed, solving for the odd terms; repeat until converged. The second derivative matrix of the cost is

$$\frac{\partial^2}{\partial b_j \partial b_i} = \begin{cases} 2\alpha & ; \quad i = j \\ -\beta & ; \quad |i - j| = 1 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

This is negative-definite, which ensures that the solution is a global optimum.

Figure (4) shows the iteration of the above equations for this uniform pdf case, allowing for different running and collecting velocities, v_r and v_c, and different numbers of intervals m. The resulting cost is shown in the Figure for $vc = 1$, with v_r ranging from 2 through 50, and m between 1 and 8. We note that for small v_r the two intervals solution is best (optimizing with more intervals leads to degenerate solutions - some of the intervals having zero width); only when the running velocity becomes quite high do many intervals provide additional gain. However, as mentioned earlier, high values of v_r are possible when the UUV transmits data by wireless connection to the base station. The dotted line in Figure (4) represents the solution with $v_r = 1$. This case represents the lower bound for the mission time, and as the number of segments increases, the mission time asymptotically approaches 0.5, which is the expected mission duration when the UUV is assumed to be capable of transmit- ting data back real-time, as is the case when using a Remotely Operated Vehicle (ROV) connected by cable to the host ship.

4 Future Work

This paper has presented an initial analysis of search operations using UUVs. Solutions to the models developed here show that the choice of parameter settings for planning the search missions can expedite the mission times considerably. Much of the development involving UUVs has, so far, been on stabilizing the technology used for its components - this work underscores the need for the construction of operational models and concepts as well. In the future, these can be extend the capabilities of UUVs and to garner insights and guide directions in technology development for these vehicles.

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