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February 2007

Originally published in:

Journal of the Acoustical Society of America, Vol. 121, No. 1, pp. 108-119, January 2007

URL: http://scitation.aip.org/dbt/dbt.jsp?KEY=JASMAN&Volume=121&Issue=1

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Separability of seabed reflection and scattering properties in reverberation inversion

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(Received 19 July 2006; revised 4 October 2006; accepted 10 October 2006)

Separation of scattering properties (Lambert's μ) from reflection properties (the reflection loss' angle derivative α) presents difficulties in the geoacoustic inversion of long range reverberation in isovelocity water, and here it is shown that there is still a problem in a refracting environment. An alternative technique is proposed where reverberation is modified by altering the source or receiver beam pattern, for instance, using a triplet array or ring source, to provide a dipole and monopole pattern. Combinations of these two measures of reverberation then conveniently determine α and μ independently of other unknown quantities from long (or short) range data, in fact even from a single range. In addition the short range ratio of the two quantities determines the critical angle independently. The effects of refraction and other source or receiver beam patterns, including a horizontal beam and a tilted beam, are investigated by using analytical techniques. To enhance the credibility of these findings and demonstrate the benefits of the approach an example is posed as a standard inversion problem using a cost function based on both types of reverberation. Finally the technique is applied to some experimental data by forming simultaneous monopole and dipole beams in the vertical plane. © *2007 Acoustical Society of America.* [DOI: 10.1121/1.2384966]

PACS number(s): 43.30.Pc, 43.30.Gv [RCG] Pages: 108–119

I. INTRODUCTION

The idea of simultaneously separating out reflection and scattering properties from reverberation has developed over 15 years or so. It has potentially important consequences for sonar performance since it can make predictions not only of both types of seabed property but predictions over a large area at considerable range from the source ship. Amongst the earliest advocates are Zhou *et al.* (1982), Kamminga, Ellis, and Gerstoft (1993), Zhou *et al.* (1993), and Gerstoft and Ellis (1996) where scattering was assumed to follow Lambert's law. Preston (2000) and Ellis and Preston (1999) have analyzed Mediterranean reverberation extensively using manual inversion techniques, and Zhou and Zhang (2003) have analyzed data from the China Sea. Simulation studies include Muller *et al.* (2002), and Makris (1993), and extension by Rogers, Muncill, and Neumann (1998) to bistatics. Reverberation data from an exercise was selected as one of the test cases for the Geoacoustics Inversion Techniques workshop in May 2001 (Fulford, King, and Chin-Bing, 2004; and Gerstoft *et al.*, 2003). Holland (2005) investigated the effect on geoacoustic inversion of assuming a scattering law other than Lambert's, and Harrison (2005a) has attempted to determine the scattering law from simultaneous reverberation and propagation measurements. In addition there are data-model comparisons, for instance, by Desharnais and Ellis (1997), and variants combining reverberation with other information sources such as the direct blast by Heaney (2003).

At first sight one might expect the effects of scattering strength and boundary reflection to be quite distinct and separable since the first is a simple multiplier while the second, one might think, produces a decay in range. On the other hand most numerical inversion techniques rely on a given physical model with implicitly prechosen search parameters, and simply demonstrate the degree of mismatch between measurements and model. Dosso (2002) and Dosso and Nielsen (2002) have proposed an approach to quantify uncertainties stemming from the mismatch. Nevertheless, some physical intuition helps to make the initial choice of input parameters in any given inversion and to avoid unwanted correlations between them (Collins and Fishman, 1995). If one initially assumes a scattering law angle dependence of the form $\mu F(\theta_1, \theta_2, \phi)$, with known F but unknown constant μ (the three arguments defining incoming and outgoing grazing angles and bistatic angle) and one assumes that, of all the boundary interactions along a round-trip reverberation path, only one is a scatter, then it is inevitable that μ will indeed be merely a multiplier of the reverberation. So to be sure of separability one needs to be certain that the reflection parameter(s) are definitely *not* simply multipliers. Some intuition on this behavior can be gathered by applying analytical techniques to propagation and reverberation.

An earlier paper by Harrison (2003a) considered isovelocity water combined with Lambert's law and showed, by using analytical techniques, that beyond a certain range, within the so-called mode-stripping region, no such separation should be possible since the reverberation intensity was proportional to $\mu\alpha^{-2}r^{-3}$, where μ is the Lambert constant, α is the reflection derivative with angle (for small angles), and r is reverberation range (i.e., corrected travel time). In other words, in this limit, one could only determine $\mu\alpha^{-2}$, not μ and α separately. This point was recently taken up by Holland (2006) who argued that although there was little hope

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for extracting unique geoacoustic reflection properties from reverberation under these conditions it would still be possible to investigate or map their variability. In passing, we note that the inseparability results from any scattering law of the form $S(\theta_1, \theta_2, \phi) = \mu \times (\theta_1 \theta_2)^n f(\phi)$, and it is not just a peculiarity of Lambert's law.

The formulas for the isovelocity case were extended to uniform sound speed gradient (Harrison, 2003b, Harrison, 2005b) and showed a more complicated behavior with respect to α and μ although the tendency to inseparability was still manifest. In Sec. II we demonstrate that even with a uniform sound speed gradient an increase in μ can be approximately matched by changing α, and exactly matched by changing α and the sound speed gradient c' at the same time. From a geoacoustic inversion point of view this is a statement that given experimental reverberation with poorly known sound speed gradient c' there is an ambiguity or correlation between μ and α (and c'). If however c' is known (and not zero) then the ambiguity is weaker. In other words, the exact shape of the decay in range is unaffected by μ (since it is just a multiplier) but it is slightly affected by α. On the other hand if, in changing α, we were to keep ($\alpha \times c'$) constant then the decay shape would also be unaffected. These findings are confirmed with a modified version of the wave model C-SNAP (Ferla, Porter, and Jensen, 1993).

The main point of this paper is to suggest an alternative and in Sec. III we propose a way of separating μ and α, still from measurements of reverberation alone, even at long range and possibly a single ping, depending on configuration. The reason for the inseparability of μ and α in conventional long range reverberation is that in the mode-stripping propagation regime α becomes a simple multiplier (see Harrison, 2003a) and is indistinguishable from μ. Because the propagation is directly dependent on the angle distribution of the multipath rays (or modes) it is affected not only by the scattering law (or kernel) angle dependence but also by the directivity of the source and receiver. By changing one or both of these directivities one can modify the propagation range dependence *in situ*, and the result is a modified reverberation. Given a measurement of "conventional" reverberation with a point source and receiver, and a measurement of "modified" reverberation with some given directivity one can deduce α alone from the relative range laws. The value of $\mu\alpha^{-2}$ derived from the reverberation then permits a separation of μ. The theory can already be extended to range-dependent and refracting environments and different scattering laws. Section IV discusses the robustness of the findings in this context.

Numerical geoacoustic inversion by matching measurements with a coherent propagation and reverberation model is a powerful technique that can take advantage of aspects of reverberation behavior that are not reflected by the incoherent diffuse reverberation behavior. We might still expect better separation performance with the added information from the modified reverberation. In Sec. V we reinforce these arguments by again using the wave model C-SNAP to apply standard inversion techniques to simulated data to investigate parameter correlations and cost function minima both with and without the modified reverberation input, in the latter case achieving a much better performance.

Finally some experimental demonstrations with a steered line array are given in Sec. VI, and the reflection and scattering parameters extracted. The resulting values of α agree with other estimates and the μ is within the expected bounds.

II. REVERBERATION DEPENDENCE ON SCATTERING, REFLECTION, AND REFRACTION: CORRELATIONS

A. Isovelocity

Harrison (2003a) provided formulas for propagation, reverberation, and target echo in isovelocity, range-dependent environments with Lambert's law scattering. Treating the eigenrays or modes as a continuum in angle their angle distribution is Gaussian and they decay exponentially with the range. In a range-independent environment these formulas naturally demonstrate three clear propagation regimes according to range, already noted by Weston (1971). In the first, the Gaussian is very wide so the angle range is limited by the critical angle, and there are many contributing rays, all about the same strength. In the second, there is "mode-stripping" and the resulting Gaussian angle distribution is narrower than the critical angle. In the third, the Gaussian is so narrow that only one mode remains. These regimes can also be seen in incoherent mode solutions where the angle distribution is similarly truncated by the critical angle. In fact there is also a zeroth regime at very short range, of little interest in this context, and predicted neither by the formulas nor by discrete mode theory, where the direct path propagation follows an inverse square law. In the context of geoacoustic inversion of long- or short-range reverberation, only two of these regimes and one transition are of interest. An appropriate formula for isovelocity reverberation (Harrison, 2003a) is

$$I = \frac{\mu \Phi p}{\alpha^2 r^3} \left\{ 1 - \exp\left(\frac{-\alpha r \theta_c^2}{2H} \right) \right\}^2, \tag{1}$$

where μ is the Lambert constant, α is related to the derivative of reflection loss with angle α_{dB} (dB/rad) through $\alpha = \alpha_{\mathrm{dB}}/(10 \log_{10}(e))$, θ_c is critical angle, H and r are water depth and range, Φ is the horizontal beam width, and p is the spatial pulse length $p = ct_p/2$. At long range this becomes

$$I = \frac{\mu \Phi p}{\alpha^2 r^3} \tag{2}$$

and at short range, expanding the exponential to first order, we have

$$I = \frac{\mu \Phi p \theta_c^4}{4 r H^2}. \tag{3}$$

The transition from short- to long-range occurs when the exponential term becomes negligible. An exact transition range can be defined by equating the neighboring limiting formulas (see Appendix)

$$r_o = \frac{2H}{\alpha\theta_c^2}. \tag{4}$$

The inseparability of μ and α at long range is seen explicitly in Eq. (2) where reverberation depends on μ/α^2. In passing we note that in principle the short-range reverberation could be used to separate the two since it has no α dependence. However, poor definition of the regimes combined with spatial interference effects, excluded by these formulas, usually rule this out as a practical option.

B. Refraction

Since formulas are also available for the case of a uniform sound speed gradient (Harrison, 2003b; Harrison, 2005b) it is interesting to check the effect of refraction on separability. In a range-independent environment there are now two distinct sets of contributing propagation angles for each range: in the first set rays interact with only the boundary on the low sound speed side; in the second set they interact with both. For the same reason there are two sets of reverberation with different behavior, one from the low sound speed side of the duct, the other from the high sound speed side. We use L and H subscripts to denote quantities associated with the low and high speed boundaries. The sound speed gradient c' is embedded in the ray's radius of curvature $\rho = c/c'$. For monostatic sonar the formulas are:

Low sound speed side:

$$I_L = (I_{L1} + I_{L2})^2 r\Phi p, \tag{5}$$

where

$$I_{L1} = \frac{\mu_L^{1/2}}{2r\rho}\log\left(\frac{H}{h}\right)\exp\left(\frac{-\alpha_L r}{2\rho}\right), \tag{6}$$

where h is the greater of the distance from source, receiver, or scatterer from the low sound speed boundary,

$$I_{L2} = \frac{2\mu_L^{1/2}}{r^2(\alpha_L + \alpha_H)}(A + B)\exp\left(\frac{-(\alpha_L - \alpha_H)r}{4\rho}\right) \tag{7}$$

with

$$A = \frac{1}{2}\left\{\exp\left(\frac{-(\alpha_L + \alpha_H)r}{4\rho}\right) - \exp\left(\frac{-(\alpha_L + \alpha_H)r\theta_c^2}{2H}\right)\right\}, \tag{8}$$

$$B = \frac{(\alpha_L + \alpha_H)r}{8\rho}\left\{E_1\left(\frac{(\alpha_L + \alpha_H)r}{4\rho}\right) - E_1\left(\frac{(\alpha_L + \alpha_H)r\theta_c^2}{2H}\right)\right\}, \tag{9}$$

where $E_1(x)$ is the exponential integral (Abramowitz and Stegun, 1972).

High sound speed side:

$$I_H = I_{H2}^2 r\Phi p, \tag{10}$$

where

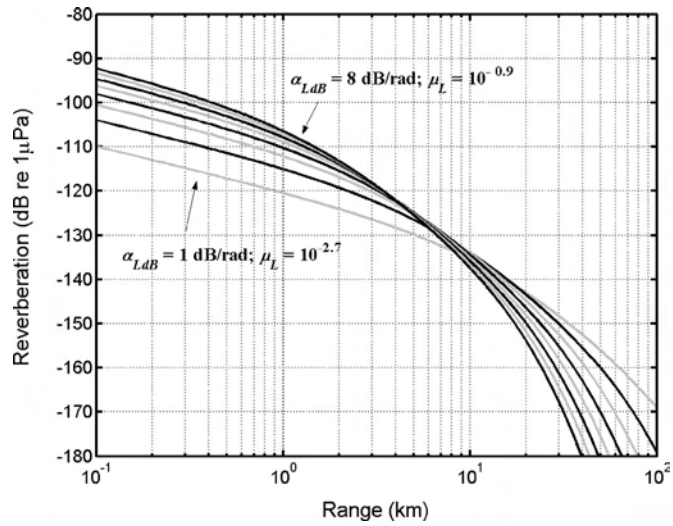

FIG. 1. Family of reverberation curves with reflection loss derivative α_{LdB} taking uniform steps between 1 and 8 dB/rad and simultaneously $\mu_L = 10^{-2.7} \times \alpha_{LdB}^2$. Other variables are fixed: $H = 100$ m, $h = 50$ m, $\theta_c = 20°$, $c_{\min} = 1500$ m/s, $c_{\max} = 1520$ m/s.

$$I_{H2} = \frac{2\mu_H^{1/2}}{r^2(\alpha_L + \alpha_H)}(A - B)\exp\left(\frac{-(\alpha_L - \alpha_H)r}{4\rho}\right). \tag{11}$$

The magnitudes of the first and second exponents in Eqs. (8) and (9) are important. Those containing the seabed's critical angle θ_c are usually much larger than those containing ρ because the equivalent to θ_c in the latter terms is $\sqrt{H/(2\rho)}$, which is half the critical angle in the water column. Thus in the long-range limit the terms containing θ_c are large, making the second exponential and the exponential integral vanish. However the other exponents may, or may not be large. Note that these formulas revert to the isovelocity case when $\rho \to \infty$ since $B \to 0$ and I_{L1} vanishes.

Clearly the relationship between μ, α, and r is not so simple with refraction even if we single out reverberation from the L side. Although μ_L is still a multiplier α_L is not. This can be demonstrated numerically by plotting Eq. (5) for various values of α_L as in Fig. 1. Here α_{LdB} is varied in uniform steps between 1 and 8 dB/rad $[\alpha = \alpha_{\mathrm{dB}}/(10\log_{10}(e)]$ but with $\mu_L = 10^{-2.7} \times \alpha_{LdB}^2$ simultaneously, while other parameters are constant $\alpha_H = 0$ dB/rad, $\mu_H = 0$, $\theta_c = 20°$, $H = 100$ m, $h = 50$ m, $c_L = 1500$ m/s, and $c_H = 1520$ m/s. For clarity source strength and Φp are set to unity throughout. In effect, Φ is one radian and p is one meter, corresponding to a 1.3 ms pulse duration. A closer inspection of the formulas Eqs. (6)–(9), however, shows that if surface losses are small, $\alpha_H = 0$, then in the long-range limit the subformulas can all be written in the form,

$$I = \frac{2}{r^2}\left(\frac{\mu}{\alpha^2}\right)^{1/2} \times F\left(\frac{\alpha r}{\rho}\right). \tag{12}$$

(In the case of I_{L1}, one needs to multiply the numerator and denominator by $\alpha_L r$.) So if ray radius of curvature ρ is maintained proportional to α_L the form of the reverberation range dependence is invariant as α_L is changed. Therefore it is always possible to match a change in scattering constant, μ_L, exactly with a change in reflection and sound speed proper-

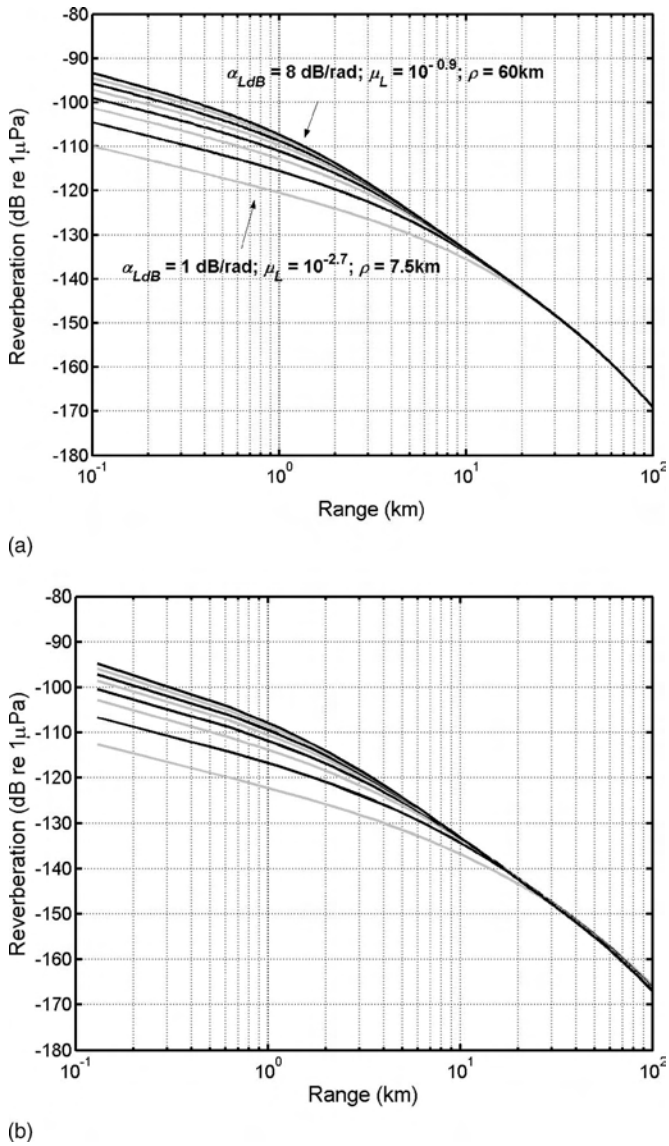

FIG. 2. Family of reverberation curves as in Fig. 1 but with μ_L and c_H simultaneously varied such that $\mu_L = 10^{-2.7} \times \alpha_{LdB}^2$ and the ray radius of curvature $\rho = 7500 \times \alpha_{LdB}$. Note the convergence at long range. (a) closed-form solution; (b) modified C-SNAP.

ties, α_L and ρ. From the geoacoustic inversion point of view one expects a strong correlation between μ_L and α_L and c'. This effect is shown in Fig. 2(a), where variation of reverberation with α_L keeping α_L/ρ constant shows clear convergence, i.e., invariance, at long range. In fact, numerically α_{LdB} again takes values 1–8 dB/rad while μ_L is set to $10^{-2.7} \times \alpha_{LdB}^2$ and c_H is varied so that $\rho = 7500 \times \alpha_{LdB}$.

C. Numerical confirmation of closed-form solutions

The findings of the previous section are here reinforced by using an independent wave model to calculate reverberation from an incoherent mode sum under the same conditions. The model uses C-SNAP (Ferla, Porter, and Jensen, 1993) operating at 1 kHz to calculate the mode shapes and horizontal wave numbers, with a separate module to do the mode summation, conversion of mode to ray angle, and calculation of group velocities, etc., according to Ellis (1995). To ensure compatible environmental inputs the seabed was composed of a half-space for which the bottom reflection loss can be calculated from the geoacoustic parameters through a relationship noted by Weston (1971), namely,

TABLE I. Baseline half-space parameters.

	Sound speed c (m/s)	Density d (kg/m^3)/1000	Vol. absorption a (dB/wavelength)
Water	1500	1.0	0.0
Sediment	1600	1.5	0.4

$$R_{\mathrm{dB}} = \alpha_{\mathrm{dB}} \times \theta = \frac{a_2}{\pi} \frac{d_2 c_1^2}{d_1 c_2^2 \sin^3 \theta_c} \times \theta,$$

where a_2 is the sediment volume absorption in dB/wavelength, and d, c are density and sound speed with subscripts 1, 2 referring to the upper and lower media. The actual geoacoustic properties corresponding to $\alpha_{\mathrm{dB}} = 3.98$ dB/rad, ($\alpha = 0.916$ rad^{-1}) are given in Table I. Other values of α_{dB} were mimicked by adjusting the volume absorption in proportion. As shown in Fig. 2(b) the results are virtually identical to the analytical examples in Fig. 2(a). The minor discrepancies reduce to a fraction of a dB throughout if one reverts to the variant of the analytical formula that includes the focusing or caustic effect dictated by the WKB depth dependence (Harrison, 2003b; Weston, 1980). In any case, the main point is to obtain a second opinion on the predicted long range convergence with a correlated set of α, μ, and sound speed gradient, and Fig. 2(b) leaves no doubt.

III. MODIFIED REVERBERATION

Harrison (2003a) assumed the propagation to follow mode-stripping and the scattering to follow Lambert's law. Other assumptions are possible, but one might expect that during any one experiment in one locality the scattering law and the propagation would be given and fixed, therefore one has no control over the reverberation. In fact there are various controls, for instance, in a refracting environment the source and receiver depth alter the results. Similarly placing either source or receiver near the sea surface creates a dipole directionality which alters the propagation by introducing an extra square-of-grazing-angle term in one or both of the outward and return angle integrals. In general, the waveguide imposes a near-Gaussian angle distribution which is multiplied by the directivities at the two ends of each propagation leg, i.e., source and scatterer for outward, and scatterer and receiver for return. There are therefore many ways one could alter the propagation integral *in situ* given the locally fixed scattering law. Some possibilities include:

- *modify source* (1): with a multiring source use one ring for the "point" source; three rings for a more directional source;
- *modify source* (2): run two rings in antiphase creating a dipole source;
- *modify source* (3): place the source near the sea surface to create a dipole (possibly problematic because of physical size and efficiency near surface);

- *modify receiver*: modify steering of a horizontal triplet array to form vertical dipoles;
- *measure angle dependence of reverberation*: use the vertical array to derive α directly from the width of the Gaussian angle distribution.

In the following we investigate three possibilities, all involving some vertical aperture, reverting to the isovelocity environment for simplicity. First the dipole, enhancing steep angles, second a horizontal beam, weakening steep angles, and third a tilted beam. In all cases we consider application to the return path only, although it is easy to extend this to the two-way path. To track the changes we note that Eq. (1) could have been written in terms of the outward and return integrals L_O, L_R as

$$I = \frac{\mu \Phi p}{rH^2} L_O L_R, \tag{13}$$

where

$$L_O = L_R = \int_0^{\theta_c} \theta \exp\left(-\frac{\alpha \theta^2 r}{2H}\right) d\theta. \tag{14}$$

We now leave L_O unchanged and calculate the new L_R as required. Introducing the shorthand

$$X = \frac{\alpha \theta_c^2 r}{2H} \tag{15}$$

we write L_R as

$$L_R = \frac{\theta_c^2}{2X}(1 - e^{-X}). \tag{16}$$

The short- and long-range limits of L_R are, respectively, $\theta_c^2/2$ and $\theta_c^2/2X$. We emphasize that we do not advocate these formulas as serious contenders for inversion directly. Rather they point the way to dependencies or correlations which can be utilized in numerical inversion by choice of the search parameters and combinations of them.

A. Vertical dipole

Assuming the dipole to be composed of two unit monopoles a distance z apart, there is a gain factor of $k^2 z^2 \sin^2 \theta$ which we approximate as $k^2 z^2 \theta^2$,

$$L_{RD} = k^2 z^2 \int_0^{\theta_c} \theta^3 \exp\left(-\frac{\alpha \theta^2 r}{2H}\right) d\theta$$
$$= k^2 z^2 \theta_c^4 \left(\frac{1 - (1 + X)\exp(-X)}{2X^2}\right). \tag{17}$$

Thus the ratio of dipole to monopole reverberation F is

$$F = \frac{I_{\text{dipole}}}{I_{\text{monopole}}} = \frac{L_{RD}}{L_R} = k^2 z^2 \frac{\theta_c^2 (1 - (1 + X)\exp(-X))}{X(1 - \exp(-X))}. \tag{18}$$

For long range ($X > 1$)

$$F = k^2 z^2 \frac{2H}{\alpha r}. \tag{19}$$

For short range ($X < 1$)

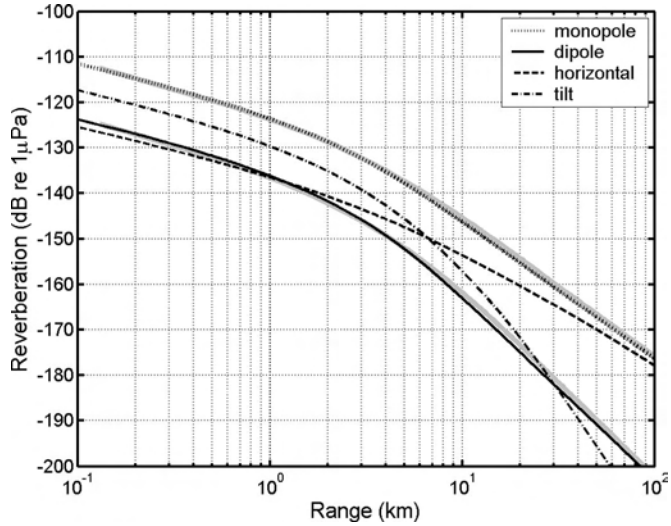

FIG. 3. Various reverberations: Monopole reverberation from the closed-form solution (dotted black) and C-SNAP (thick grey); dipole reverberation from the closed-form (solid black) and C-SNAP (thick grey); horizontal beam closed-form (dashed black); tilted beam closed-form (dashed-dotted black).

$$F = k^2 z^2 \theta_c^2 / 2. \tag{20}$$

The full behavior is shown as the solid black line in Fig. 3 (suppressing the $k^2 z^2$ term) for the baseline parameters ($H = 100$ m, $\mu = 10^{-2.7}$, $\alpha_{\text{dB}} = 4$ dB/rad, $\theta_c = 20°$, see Table I) and compared to the conventional monopole reverberation (dotted line). The equivalent ratio F is shown in Fig. 4. The most important point is that because of Eq. (19) the range dependence is completely altered in a very simple way at long ranges. In fact, the ratio of this modified reverberation to conventional reverberation *F is independent of μ* at all ranges, as is clear in Eq. (18). If we plot $r \times F$ as shown in Fig. 5 we expect a long range plateau with value of $2H/\alpha$. This means that if one measures monopole and dipole reverberation, then α and μ are separable after all. In fact in this example the value of the plateau is 23.37 dB

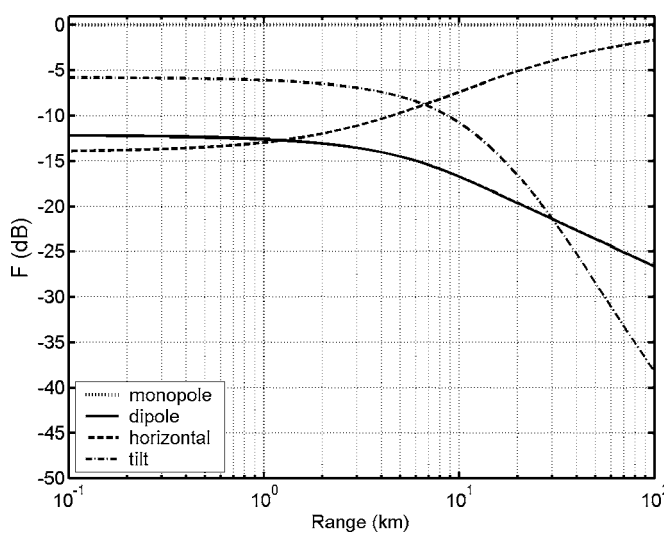

FIG. 4. The closed-form ratio (function F) of the quantities in Fig. 3 to monopole reverberation. Monopole (dotted); dipole (solid); horizontal beam (dashed); tilted beam (dashed-dotted).

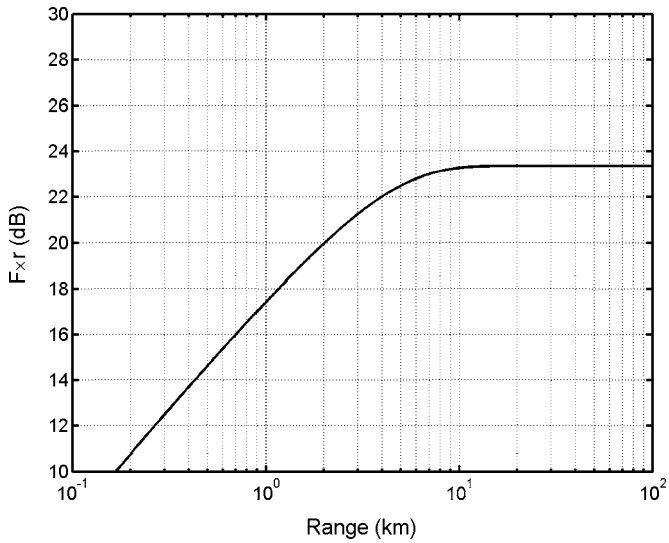

FIG. 5. The ratio multiplied by range $F\times r$ showing the plateau at long range.

which leads to $\alpha_{\mathrm{dB}}=3.9977$ dB/rad and agrees with the input value. Having separated α one can deduce μ from long range monopole reverberation. Although of less practical interest, the short range plateau in F (solid line in Fig. 4) occurs at −12.19 dB, which according to Eq. (20) gives an independent estimate of the critical angle. The estimate is $\theta_c=19.9°$ which again agrees with the input value. Despite the inaccuracy of the formulas in the presence of spatial interference, these findings suggest that a more sophisticated model of the ratio F would still be sensitive to α alone at long range and θ_c alone at short range. Notice that it is the nature of the function in Eq. (18) that the short range to long range transition "knee" is about two times further out in range than the "knee" in Eq. (1). This is a peculiarity of the function as explained in the Appendix. Numerical confirmation of these findings for the monopole and dipole is shown by the thick grey lines in Fig. 3 for the same parameters (determined as discussed in Sec. II C). Agreement is excellent.

B. Horizontal beam

A horizontal beam is conveniently represented as the Gaussian $\exp(-\theta^2/\theta_B^2)$. Although the effect on the integral is trivial the effect on reverberation is more interesting since it affects range dependence [through Eq. (15)],

$$L_{RH}=\int_0^{\theta_c}\theta\exp\left(-X\frac{\theta^2}{\theta_c^2}-\frac{\theta^2}{\theta_B^2}\right)d\theta$$
$$=\frac{\theta_c^2}{2\left(X+\frac{\theta_c^2}{\theta_B^2}\right)}\left\{1-\exp\left(-\left(X+\frac{\theta_c^2}{\theta_B^2}\right)\right)\right\} \tag{21}$$

and the ratio of reverberation obtained with a horizontal beam and with an isotropic source is

$$F=\frac{L_{RH}}{L_R}=\frac{X}{\left(X+\frac{\theta_c^2}{\theta_B^2}\right)}\frac{\left\{1-\exp\left(-\left(X+\frac{\theta_c^2}{\theta_B^2}\right)\right)\right\}}{\{1-\exp(-X)\}}. \tag{22}$$

The transition determined by the magnitude of the exponent is now when

$$X+\frac{\theta_c^2}{\theta_B^2}\sim 1. \tag{23}$$

If the beam is narrower than the critical angle then there is essentially no transition in the exponential—it is negligible for all ranges—but there is one in the denominator. For

$$X<\frac{\theta_c^2}{\theta_B^2},\quad L_{RH}=\theta_B^2/2, \tag{24}$$

whereas for

$$X>\frac{\theta_c^2}{\theta_B^2},\quad L_{RH}=H/\alpha r, \tag{25}$$

which leads to the same r^{-3} behavior for isotropic source and receiver ($F=1$). The effect is shown in Figs. 3 and 4 for $\theta_B=4°$. In summary a narrow horizontal beam has an effect, but it is too weak to be useful for inversion.

C. Tilted beam

If the beam of width θ_B is tilted upwards at θ to the horizontal we have

$$L_{RT}=\frac{1}{2}\int_{-\theta_c}^{\theta_c}|\theta|\exp\left(-X\frac{\theta^2}{\theta_c^2}\right)\exp\left(-\left(\frac{\theta-\theta_o}{\theta_B}\right)^2\right)d\theta$$
$$=\left\{\int_0^{\theta_c}\theta\exp(-A\theta^2-B\theta-C)d\theta+\int_0^{\theta_c}\theta\exp(-A\theta^2+B\theta-C)d\theta\right\}/2, \tag{26}$$

where

$$A\equiv\frac{\alpha r}{2H}+\frac{1}{\theta_B^2}=\frac{X}{\theta_c^2}+\frac{1}{\theta_B^2};\quad B\equiv\frac{2\theta_o}{\theta_B^2};\quad C\equiv\frac{\theta_o^2}{\theta_B^2}.$$

It is possible to solve this exactly for arbitrary θ_o, θ_B, θ_c but a reasonable approximation is for large θ_c (see 3.462/5, Gradshteyn and Ryzhik, 1980),

$$L_{RT}=\frac{\exp(-C)}{2A}\left\{1+\frac{B\sqrt{\pi}}{2\sqrt{A}}\exp\left(\frac{B^2}{4A}\right)\mathrm{erf}\left(\frac{B}{2\sqrt{A}}\right)\right\}. \tag{27}$$

The corresponding reverberation and F function are shown in Figs. 3 and 4 for $\theta_B=4°$ and $\theta_o=15°$. A poorer but more insightful approximation is a narrowish beam inside the critical angle, i.e:, $\theta_B\ll\theta_c$; $\theta_o<\theta_c$,

$$L_{RT}=\frac{\sqrt{\pi}}{2}\theta_B\theta_o\exp\left(-\frac{\alpha\theta_o^2 r}{2H}\right)=\frac{\sqrt{\pi}}{2}\theta_B\theta_o\exp\left(-X\frac{\theta_o^2}{\theta_c^2}\right), \tag{28}$$

$$F = \frac{\alpha r}{H}\frac{\sqrt{\pi}}{2}\theta_B\theta_o \exp\left(-\frac{\alpha\theta_o^2 r}{2H}\right)$$
$$= \frac{\sqrt{\pi}}{2}\frac{\theta_B\theta_o}{\theta_c^2}X\exp\left(-X\frac{\alpha\theta_o^2}{\theta_c^2}\right). \quad (29)$$

In theory, this behavior is virtually the same as that of the integrand of Eqs. (26) or (14), and so it contains more information (given a variable beam angle) than, say the dipole beam. Nevertheless the simple relationship between dipole and conventional reverberation makes it more attractive.

IV. ROBUSTNESS AND EXTENSIONS

The main point of this paper, and in particular Sec. III, is to demonstrate that reflection properties can, after all, be separated out from reverberation. Since the derivations have made certain assumptions we explore their weaknesses here.

A. Range-dependence

If we assume variable bathymetry, isovelocity reverberation formulas are given by Harrison (2003a) in terms of an effective depth, conceived by Weston (1976). These have been justified and favorably compared with the model BiStaR (LePage and Harrison, 2003; Harrison, 2005c). As is apparent from the original definition, a variable α can also be accommodated as part of the effective depth integral, so the original term in the exponent $\alpha\theta_s^2\int dr/H(r)^3$ becomes $\theta_s^2\int\alpha(r)dr/H(r)^3$ (see also Holland, 2006) and we define a modified effective depth

$$\chi_{\text{eff}}(r) = \frac{H_s^2 H(r)^2}{r}\int_0^r \alpha(r)dr/H(r)^3 \quad (30)$$

in between a notional fixed source at range zero and receiver at r. Following Harrison's derivation through, and making the substitution [cf. Eq. (15)],

$$X(r) = \frac{\theta_c^2 H_c^2 \chi_{\text{eff}} r}{2H_s^2 H(r)^2} \quad (31)$$

the monopole reverberation with variable α and μ is

$$I_{\text{monopole}} = \frac{\mu\Phi p}{r^3}\left\{\frac{H_s}{\chi_{\text{eff}}}(1-\exp(-X))\right\}^2. \quad (32)$$

Following Sec. III A the dipole reverberation is

$$I_{\text{dipole}} = \frac{\mu\Phi p}{r^3}\left\{\frac{H_s}{\chi_{\text{eff}}}(1-\exp(-X))\right\}$$
$$\times\left\{\frac{H_s}{\chi_{\text{eff}}}(1-(1+X)\exp(-X))\right\}\frac{2H(r)^2}{r\chi_{\text{eff}}} \quad (33)$$

and the ratio of the two (suppressing the k^2z^2) is

$$F(r) = \frac{2H(r)^2}{r\chi_{\text{eff}}}\frac{(1-(1+X)\exp(-X))}{(1-\exp(-X))}. \quad (34)$$

Since range and bathymetry are assumed known, the ratio F at long range tells us χ_{eff} (r), and from its definition, Eq. (30), range-smoothing followed by differentiation of the quantity $\chi_{\text{eff}}r/(H_s^2H(r)^2)$ gives $\alpha(r)/H(r)^3$. Therefore the range-dependent $\alpha(r)$ can be computed directly, in principle. In this same long range limit, knowing $\alpha(r)$ and $H(r)$ one can then compute $\mu(r)$ from the absolute value of I_{monopole} at each range. Note that θ_c, whether range-dependent or -independent is not obtainable in the long range limit.

B. Scattering law

From Eqs. (13) and (14) it is clear that an angle-separable scattering law of the more general form $S(\theta_{\text{in}},\ \theta_{\text{out}})=\mu(\theta_{\text{in}}\times\theta_{\text{out}})^n$ can be accommodated in the derivations of Sec. III. The ratio of dipole to monopole reverberation in the long range limit becomes

$$\frac{I_{\text{dipole}}}{I_{\text{monopole}}} = \frac{\int_0^\infty \theta^{n+2}\exp(-a\theta^2)d\theta}{\int_0^\infty \theta^n\exp(-a\theta^2)d\theta}, \quad (35)$$

where a is a constant, representing some function of α, H, r, etc., and changing the variable to $Z=a\theta^2$ we have

$$\frac{I_{\text{dipole}}}{I_{\text{monopole}}} = \frac{1}{a}\frac{\int_0^\infty Z^{(n+1)/2}\exp(-Z)dZ}{\int_0^\infty Z^{(n-1)/2}\exp(-Z)dZ}$$
$$= \frac{1}{a}\frac{\Gamma(1+(1+n)/2)}{\Gamma((1+n)/2)} = \frac{1}{a}\times\frac{(1+n)}{2} \quad (36)$$

for any positive value of n (not necessarily integer) where Γ is the gamma function (Abramowitz and Stegun, 1972). We obtain the same a from the ratio, whether as in Eq. (19) or the range-dependent version, Eq. (34), but with a small change in the scaling factor of $(1+n)/2$, i.e., 1 for Lambert's law, 0.75 for Lommel-Seeliger (or square root equivalent, see Holland, 2005), 0.5 for angle-independent. Thus the measure of α is relatively insensitive to discrepancies between the actual and assumed scattering law. However one expects the assumption of Lambert in the presence of a lower angle power in reality to lead to an overestimate of α.

In the extreme case of a nonangle-separable scattering law $S(\theta_{\text{in}},\theta_{\text{out}})=\mu f(\theta_{\text{in}},\theta_{\text{out}})$ one can still say several things. First, $f(\theta_{\text{in}},\theta_{\text{out}})=f(\theta_{\text{out}},\theta_{\text{in}})$. Second, the functional form of f will clearly affect the result. But, most importantly, the dipole to monopole ratio does not depend on the strength of the scattering, μ, since it cancels. It therefore *does* depend on reflection properties modulated by the function f.

C. Refraction

The effects of refraction on reverberation were explored in Sec. II B, Eqs. (5)–(11). Of interest here is the effect of refraction on the long range ratio F. First, consider orders of magnitude. In the long range limit the second terms in Eqs. (8) and (9) tend to zero. For strong refraction we might have $\rho=c/c'\sim 1500\times 100/20=7.5$ km and $\alpha=0.5$ rad^{-1} so that the decay distance for the first exponential in Eq. (8) (i.e., $4\rho/\alpha$) would be 60 km. In a rough calculation we can ne-

glect the B term since it is proportional to the *difference* between reverberation from the low sound speed and the high sound speed side of the duct. Equations (6) and (7) reduce to

$$I_{\text{monopole}} = \frac{\mu\Phi p}{r^3\alpha^2}\left(\frac{\alpha r}{2\rho}\log\left(\frac{H}{h}\right)+1\right)^2 \exp(-\alpha r/\rho). \tag{37}$$

The ray angle at the array is always in between that at the sea surface and seabed, but since we have already neglected the difference, it is straightforward to form I_{dipole} by inserting a θ^2 term in the integrals that resulted in Eqs. (6) and (7). The result is

$$I_{\text{dipole}} = \frac{\mu\Phi p}{r^3\alpha^2}\left(\frac{\alpha r}{2\rho}\log\left(\frac{H}{h}\right)+1\right)\left(\frac{\alpha r}{2\rho}\frac{2(H-h)}{\rho}+\frac{H}{2\rho}+\frac{2H}{r\alpha}\right) \times\exp(-\alpha r/\rho). \tag{38}$$

Writing $b \equiv \alpha r/(4\rho)$ and $\eta \equiv h/H$ the ratio is

$$F = \frac{2H}{\alpha r} \times \frac{1+b+8b^2(1-\eta)}{1-2b\log(\eta)}. \tag{39}$$

Clearly the dependence of F on H, α, and r is unchanged except for a numerical factor, which for $b=1$ and $\eta=0.5$ is 2.51. Thus the numerical inversion technique is robust in the presence of refraction.

D. Formation of the dipole beam pattern

In practice it may be difficult to produce a perfect dipole response. It goes without saying that the null needs to be as deep as possible since it is attempting to reject near horizontal paths. If there is a hint of monopole behavior of the form $B(\theta)=\delta+k^2z^2\sin^2\theta$, then the expected weak returns and rapid fall-off with range will be contaminated and the function F [Eq. (19)] will be poorly estimated.

V. STANDARD INVERSION TECHNIQUES

A. Standard geoacoustic inversion of conventional reverberation only

Here the objective is to demonstrate, by examining a cost function, that application of standard inversion techniques to conventional reverberation alone would yield poor estimates of α and μ. The reverberation was again simulated by the modified C-SNAP already described in Sec. II C for an isovelocity water column and flat bottom environment as in Table I, again varying α by adjusting the volume absorption a_2 in proportion. The baseline values of α and μ are 0.92 rad^{-1} (corresponding to $\alpha_{\text{dB}}=4$ dB/rad) and −27 dB. For demonstration purposes we deliberately avoid introducing frequency variation in these parameters; in practice there may be an additional benefit in including such variation in the cost function. Figure 6 shows the misfit ambiguity surface (least-mean-square) as a function of μ and α^2 and averaging over ranges from 3 km to 50 km. The baseline values are indicated by a plus sign. There is a clear trough along the diagonal where $\mu \propto \alpha^2$ is predicted by Eq. (2). However the bottom of the trough is not absolutely flat so it is possible to obtain a fit (indicated by the circle), though only in the absence of noise. "Noise" can mean several things in this

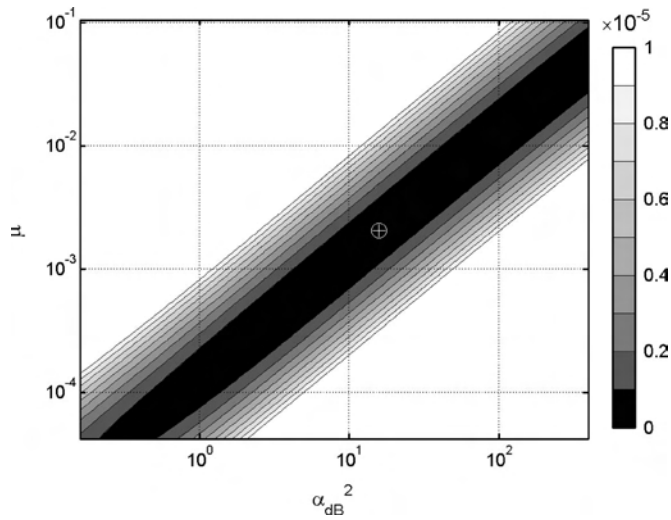

FIG. 6. Behavior of least-mean-square cost function based on monopole reverberation alone. The baseline values of α_{dB} and μ are indicated by a +, and the deduced solution by a ∘.

context. First, from a pure inversion theory standpoint, one might envisage some randomness added in the α, μ plane. Secondly ambient noise, when added to reverberation measurements only affects the longest ranges, and one therefore should, as far as possible, arrange the experiment to minimize ambient noise (see Sec. VI). Thirdly, a discrepancy between the chosen physical model and the experimental measurements may result in spatial (or temporal) fluctuations which spoil the goodness of match. Obviously the slightest disagreement between the physical model and measurements will result in arbitrary shifts predominantly along the trough. This is a manifestation of the inseparability of α and μ that this paper addresses. How the inversion algorithm copes with this trough and noise depends on which algorithm is chosen, and a separate investigation is out of the scope of this paper.

B. Simultaneous geoacoustic inversion of reverberation and dipole reverberation

With the addition of, effectively, a second measurement there are several options for processing. One possibility is to construct a joint cost function following the approach used by Nielsen (2006) where the two measurements were reverberation and propagation. In the present context where we are only searching for two parameters we have two measurements (even at a single range) and two unknowns, μ and α. If one were to assume the long range versions of Eqs. (1) and (18), namely Eqs. (2) and (19), to be true one could write down a formula for α only and a formula for μ only. Instead, for reasons that will become apparent, we *define* a quantity A

$$\mathrm{A} \equiv k^2z^2\frac{2HI_{\text{monopole}}}{rI_{\text{dipole}}} = k^2z^2\frac{2H}{rF} = \alpha \times \frac{(1-\exp(-X))}{(1-(1+X)\exp(-X))} \tag{40}$$

such that, in the long range limit it would converge on α. Similarly we define a quantity M

$$\mathrm{M} \equiv \frac{I^3_{\text{monopole}} r}{I^2_{\text{dipole}}} \frac{(2Hk^2z^2)^2}{\Phi p}$$
$$= \mu \times \left(\frac{(1-\exp(-X))^2}{(1-(1+X)\exp(-X))} \right)^2 \tag{41}$$

that in the long range limit would converge on μ. From the point of view of numerical inversion the two parameters A and M are already orthogonal in the long range limit and do not require any coordinate rotations (see Collins and Fishman, 1995). Their orthogonality suggests the following joint cost function,

$$E(\alpha_S, \mu_S) = \sum_i (\mathrm{A}_i - \mathrm{A}'_i(\alpha_S, \mu_S))^2 + w$$
$$\times (\mathrm{M}_i - \mathrm{M}'_i(\alpha_S, \mu_S))^2, \tag{42}$$

where A, M denote measurements and A', M' denote modeled quantities, w is a relative weighting found by trial and error, and the summation is assumed to be over a selection of ranges. Figure 7 is a plot of E as a function of the search parameters α_S, μ_S for ranges between 3 km and 50 km. The first and second terms of Eq. (42) are shown individually in Figs. 7(a) and 7(b), so that one can see the closeness to orthogonality of the functions, then combined in Fig. 7(c). The improvement over Fig. 6 is dramatic, and there is a well defined solution for both parameters which agrees with the simulated experimental values of $\alpha=0.92$ and $\mu=10^{-2.7}$ as shown by the plus symbol in the figure. In fact the fit is still good for shorter ranges, and the reason for this is that the function of X on the right-hand side of Eq. (41) tends to a constant (actually 2^2) for small X as well as tending to a constant (1) for large, so numerically μ can be fixed. Because μ is fixed, α can also be fixed despite the fact that the function of X at the right-hand side of Eq. (40) tends to $2/X$ leading to the whole right-hand side being a constant, independent of α. By a useful quirk, that constant $4H/(\theta_c^2 r)$ depends only on the critical angle.

In retrospect it is clear from Eqs. (40) and (41) that a measurement of monopole and dipole reverberation at even a single range (given H, z, etc.) is enough to define A and M and therefore α and μ if that range is long. For instance, in this simulated example (or Fig. 3) we have $I_{\text{monopole}}=155.3$ dB and $I_{\text{dipole}}=174.9$ dB at 20 km. This leads to $\alpha=0.91$, $\mu=10^{-2.7}$ again agreeing with the simulated experimental values. Obviously the sensitivity to the two absolute reverberation levels requires some kind of averaging, but not necessarily over range. Although this appears to obviate numerical inversion altogether, in a more realistic inversion one might be searching for more than just two parameters (for instance, critical angle or geoacoustic layer parameters) so one can justify using the more robust standard inversion technique.

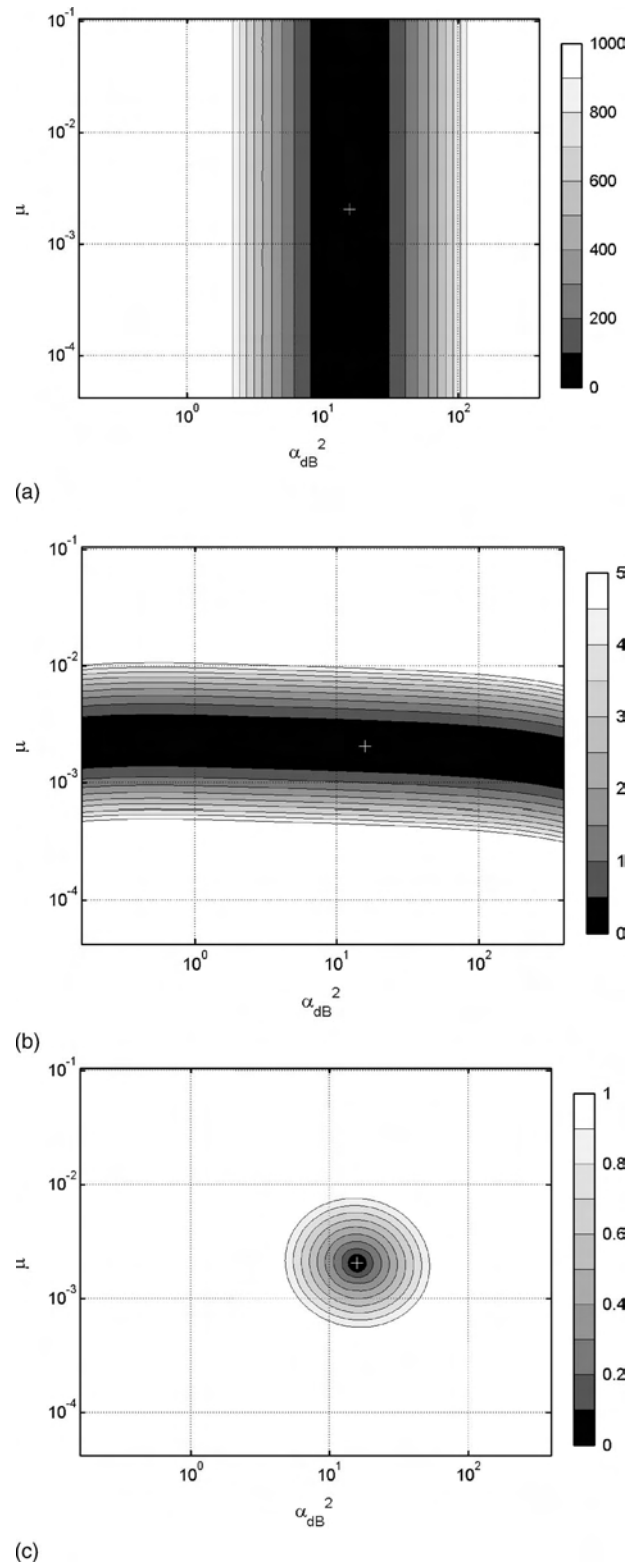

FIG. 7. Behavior of the first (a) and second (b) quasiorthogonal term in the cost function [Eq. (42)], and the joint cost function (c) derived from monopole and dipole reverberation. Baseline values are indicated by a +.

VI. EXPERIMENTAL DETERMINATION OF α FROM THE REVERBERATION RATIO

An ideal dipole receiver would be a towed horizontal triplet array since it is readily available as a means of resolving the left/right ambiguity by cardioid beam-forming. In principle the triplets can be weighted as $[\sin(\phi); \sin(\phi+2\pi/3); \sin(\phi+4\pi/3)]$, knowing the roll angle for each triplet, to form an effective vertical dipole. Unfortunately suitable data have not been found by these authors. Instead we use here data gathered with a vertical array (VLA) to mimic a sine-squared beam since all beams are known. Of

course, in this case, one could treat each vertical beam as a "tilted beam" (Sec. III C), but the simplicity of the dipole formulas and the appeal of a dipole or triplet array compels us to demonstrate that approach. The data were collected at three sites on the Malta Plateau during BOUNDARY2003 (July 8th, 2003) and BOUNDARY2004 (May 17 and 20, 2004), and have already been described in Harrison (2005a). Each set consists essentially of monostatic reverberation on a gradual slope with the source and array in about 150 m water depth but with returns from water shallowing to about 80 m over some tens of kilometers. There was a strong summer profile in 2003 with a velocity contrast of 30 m/s, whereas in 2004 the gradients were much weaker with a contrast of about 10 m/s. The source was a sweep from 700 to 1500 Hz at 209 dB re 1 μPa at 1 m, and the 32-element VLA had angle resolution of a few degrees at its design frequency, 1500 Hz. Given the smoothed beam responses it is an easy matter to sum over angles (monopole) and multiply by $\sin^2(\theta)$ then sum (dipole). The match-filtered response for the three dates extending out to about 25 km are shown in Figs. 8(a)–8(c), and these can be compared with Fig. 3. The spike near 10 km on the 8th and the 20th is the Campo Vega oil rig and tender. From approximately 1 to 25 km in all cases one can see the expected divergence of the monopole and dipole curves. The weaker dipole curves tend to flatten off into ambient noise at a shorter range. Also the Campo Vega spike is weaker compared with its surrounding in the dipole case, which is to be expected since it is a predominantly horizontal return. Both the continuum of angles theory in Secs. II and III and the discrete mode theory of C-SNAP exclude angles greater than the critical angle. They therefore do not model behavior correctly at short range. However, it is interesting to note that the direct path propagation (through its angle-independence) would lead to the same formula for F [Eq. (20)] in this region. In fact the dB difference of 12 or 13 dB (which is strikingly uniform despite the spatial fluctuations of the individual curves) is close to that expected for a critical angle of about 20°.

The equivalent of Fig. 5 for all cases is shown in Fig. 9. Now one can clearly see the effects of ambient noise beyond about 10 km, and the fall-off at short range, leaving a plateau in the middle at 23, 21, 21 dB, respectively. If a flat bottom and Lambert's law were assumed with corresponding depths of 149, 143, 165 m we would arrive at α values of 1.49, 2.27, 2.62 rad^{-1}. Allowing for the fact that the dominant reverberation is from the shallow side of the array a more realistic average depth would be 100 m, so the values of α would reduce to 1.00, 1.58, 1.58 rad^{-1}. One could also include more detailed bathymetry and azimuthal effects through an effective depth, as in Harrison (2003a) or Harrison (2005c), where three-dimensional slope effects were considered. However the effective depth with a uniform bottom slope is just the average of the endpoint depths so we would expect the above estimate to remain valid. Note that we have no reason to suspect spatial changes in α in this environment so we do not invoke Eq. (30).

Since the sound speed profiles were not isovelocity it is interesting to calculate the correction to the long range ratio F using Eq. (39). In fact $\eta = h/H \approx 0.5$ and assuming $\alpha = 1$ rad^{-1} and a (uniform) sound speed gradient of 0.1 s^{-1} the extra factor is 1.04 at 10 km. Increasing the sound speed gradient to 0.3 s^{-1} the extra factor becomes 1.5 at 10 km. These factors are close enough to unity to justify the isovelocity estimates above.

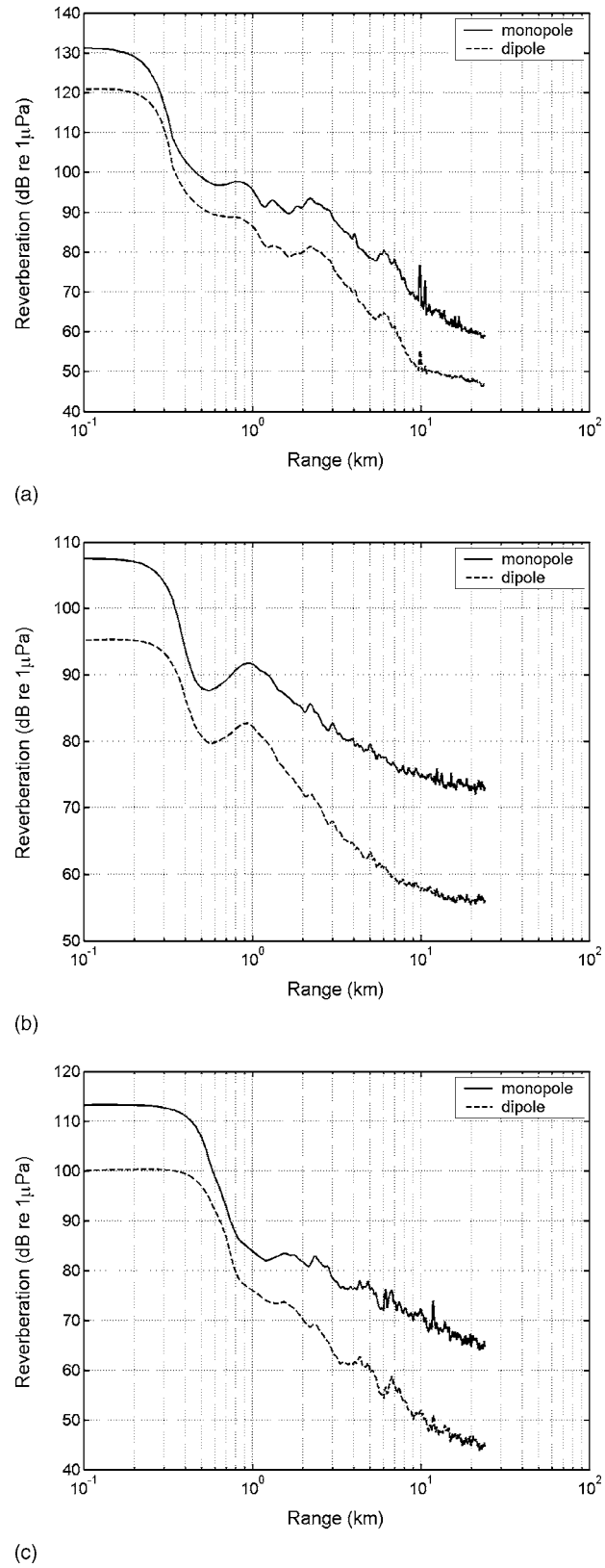

FIG. 8. Monopole (solid) and dipole (dashed) reverberation on the Malta Plateau for (a) July 8, 2003; (b) May 17, 2004; (c) May 20, 2004.

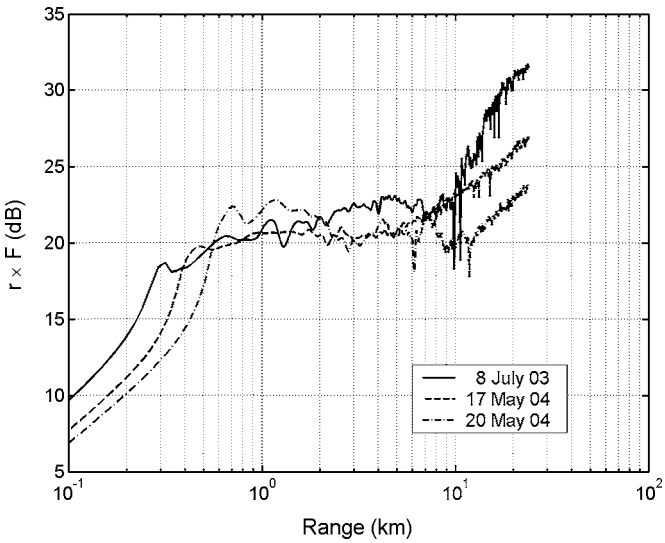

FIG. 9. The ratio of dipole to monopole reverberation multiplied by range for July 8, 2003 (solid), May 17, 2004 (dashed), May 20, 2004 (dashed-dotted).

There is evidence that the scattering law may have a weaker dependence on angle than Lambert (Holland, 2006), and it was shown in Sec. IV B that a law, such as Lommel-Seeliger, with $n=0.5$, would result in an extra factor of 0.75 in the dipole-monopole ratio. This would reduce the α values further to 0.75, 1.19, 1.19 rad^{-1}. These values are close to the mean (1.1 rad^{-1}) and well within the bounds set by an independent measurement technique, already reconciled with other measurements, for the same area based on one-way propagation paths (Prior and Harrison, 2004). In fact the upper Lambert values and the lower Lommel-Seeliger values lie on the edge of their error distributions.

Inserting $\alpha=1.1$ rad^{-1} into Eq. (1) and taking the monopole reverberation value of 68 dB at a range of 10 km, with $\Phi=2\pi$, $p=1.87$ m (1.25 ms pulse), and vertical beam width 3.5°, we find $\mu=-33$ dB. This value is close to other measurements for the area (see for instance, Holland, 2006), for which there is a wide spread, and of course this evaluation depends on the sonar calibration, unlike the evaluation of α. In addition, for comparison purposes, long range measurements of μ that are immune from propagation uncertainty are hard to come by for the reasons central to this paper.

VII. CONCLUSIONS

An analytical approach resulting in closed-form solutions has been used and extended to propose a more robust way of separating and extracting geoacoustic and scattering parameters from reverberation. From a philosophical point of view the marrying of analysis with a numerical technique has been demonstrably fruitful.

An earlier paper (Harrison, 2003a) suggested on theoretical grounds that in an isovelocity environment long range reverberation is a function of μ/α^2 (where α is the angle derivative of reflection loss and μ is the scattering coefficient). So α and μ cannot be separated by inverting reverberation alone. In this paper we have shown that in a refracting environment there is a strong correlation, i.e., relationship, between μ, α and sound speed gradient (ray radius of curvature ρ). Because of this, the shape of the reverberation curve at long range for a given μ, can always be matched by a change in α *and* ρ. Thus if ρ is predetermined then, in principle, α and μ *can* be separated since they affect the curve in different ways. However if, ρ is unknown there is enough freedom to match a change in μ with a joint change in α and ρ.

This paper has proposed an alternative geoacoustic inversion technique in which the reverberation is deliberately modified by biasing the propagation angles through a source or receiver beam, thus providing two separate measures of reverberation. The relative behavior of these two measures then provides the required separability between scattering and geoacoustic parameters. Of the various possibilities, three were investigated: a dipole beam, a narrow horizontal beam, and a tilted beam. The dipole reverberation combined with monopole (conventional) reverberation leads to some surprisingly powerful, simple, and robust results. For instance, from the ratio at long range we can find α directly and independently of other parameters (other than water depth). Having found α one can then deduce μ. The short range ratio is sensitive to critical angle and insensitive to μ and α. These deductions were made by extending Harrison's closed-form solutions, and they were confirmed using a modified version of the wave model C-SNAP. Although the choice of measurements (i.e., the ratio) was suggested by appeal to closed-form solutions this paper does not advocate using closed-form solutions for inversion.

Simulations using C-SNAP were used to test the separability of μ and α in the context of geoacoustic inversion of reverberation. A comparison was made between an approach based on monopole reverberation alone and joint monopole and dipole reverberation. The first case leads to a strong correlation between μ and α^2, as expected, and therefore the prospect of difficult separation with noisy data. The second used the theory to suggest orthogonal search parameters which then led to a very well defined minimum.

Although the simulation assumed an isovelocity range-independent environment it was shown mathematically that the approach can handle range-dependent environments and still separate α, μ in the long range limit. The approach is robust to refraction and the derived α is relatively insensitive to the assumed scattering law.

Separability of α and μ was demonstrated with experimental reverberation data from a vertical array. Since elevation angles were known it was possible to form an exact analog of the monopole and the dipole receiver beams by multiplying by, respectively, unity and sine squared of beam angle, before summing over all angles at each delay. The results clearly showed the expected divergence of the two varieties of reverberation at long range from which one could derive separately the three quantities, α, μ, and θ_c. This operation ought to, in principle, be possible with a towed horizontal triplet array steered as a vertical dipole or monopole.

ACKNOWLEDGMENTS

The authors would like to thank the Captain and crew of the *NRV Alliance* and Enzo Michelozzi, Luigi Troiano and

Piero Boni for taking care of the hardware and data acquisition during the BOUNDARY experiments.

APPENDIX: TRANSITION FROM "SHORT RANGE" TO "LONG RANGE"

Although it might seem a bit fussy to investigate as vague a phenomenon as the transition from short range to long range behavior (the "knee"), its position has important consequences for the viability of this technique in practice since it influences the lower limit of "long range" where the theory is at its best. As a working definition of a knee we take the well-defined point on a $\log(r)$ graph where the short range and long range asymptotes intersect. Taking Eq. (1), the long- and short-range limiting forms are already specified in Eqs. (2) and (3). The intersection is found by simply equating the two, thus the knee is at

$$r_o = \frac{2H}{\alpha\theta_c^2} \quad \text{or } X = 1$$

as we might have guessed. To do the same operation on Eq. (18), we equate the long and the short range asymptotes given by Eqs. (19) and (20), and we find

$$r_o = \frac{4H}{\alpha\theta_c^2} \quad \text{or } X = 2.$$

Thus the knee of the ratio of dipole to monopole reverberation occurs at twice the range of the monostatic reverberation knee. As pointed out in Harrison (2003a) the latter tends to occur at about 20 water depths for many bottom types.

Abramowitz, M. and Stegun, I. A. (**1972**). *Handbook of Mathematical Functions* (Dover, New York).

Collins, M. D. and Fishman, L. (**1995**). "Efficient navigation of parameter landscapes," J. Acoust. Soc. Am. **98**, 1637–1644.

Desharnais, F. and Ellis, D. D. (**1997**). "Data-model comparisons of reverberation at three shallow-water sites," IEEE J. Ocean. Eng. **22**, 309–316.

Dosso, S. E. (**2002**). "Quantifying uncertainty in geoacoustic inversion. I. A fast Gibbs sampler approach," J. Acoust. Soc. Am. **111**, 129–142.

Dosso, S. E. and Nielsen, P. L. (**2002**). "Quantifying uncertainty in geoacoustic inversion. II. Application to broadband, shallow-water data," J. Acoust. Soc. Am. **111**, 143–159.

Ellis, D. D. (**1995**). "A shallow-water normal-mode reverberation model," J. Acoust. Soc. Am. **97**, 2804–2814.

Ellis, D. D. and Preston, J. R. (**1999**). "Extracting sea-bottom information from reverberation data," J. Acoust. Soc. Am. **105**, 1042.

Ferla, C. M., Porter, M. B., and Jensen, F. B. (**1993**). "C-SNAP: Coupled SACLANTCEN normal mode propagation loss model," NATO Undersea Research Centre Memorandum Report No. SM-274, December 1993.

Fulford, J. K., King, D. B., Chin-Bing, S. A., and Chapman, N. R. (**2004**). "Benchmarking geoacoustic inversion methods using range-dependent field data," IEEE J. Ocean. Eng. **29**, 3–12.

Gerstoft, P. and Ellis, D. D. (**1996**). "Application of multifrequency inversion methods to obtain seabed properties from broadband reverberation data," J. Acoust. Soc. Am. **100**, 2665.

Gerstoft, P., Hodgkiss, W. H., Kuperman, W. A., and Song, H. (**2003**). "Phenomenological and global optimization inversion," IEEE J. Ocean. Eng. **28**, 1–10.

Gradshteyn, I. S. and Ryzhik, I. M. (**1980**). *Table of Integrals, Series, and Products* (Academic, New York).

Harrison, C. H. (**2003a**). "Closed-form solutions for reverberation and signal-excess with mode-stripping and Lambert's law," J. Acoust. Soc. Am. **114**, 2744–2756.

Harrison, C. H. (**2003b**). "Signal and reverberation formulae including refraction," NATO Undersea Research Centre Report No. SR-370, April 2003.

Harrison, C. H. (**2005a**). "Experimental determination of seabed scattering law and environmental parameters from reverberation," NATO Undersea Research Centre Report No. SR-403, February 2005.

Harrison, C. H. (**2005b**). "Closed form bistatic reverberation and target echoes with variable bathymetry and sound speed," IEEE J. Ocean. Eng. **30**, 660–675.

Harrison, C. H. (**2005c**). "Fast bistatic signal-to-reverberation-ratio calculation," J. Comput. Acoust. **13**, 317–340.

Heaney, K. D. (**2003**). "Active rapid geo-acoustic characterization," J. Acoust. Soc. Am. **113**, 2191.

Holland, C. W. (**2005**). "On errors in estimating seabed scattering strength from long-range reverberation (L)," J. Acoust. Soc. Am. **118**, 2787–2790.

Holland, C. W. (**2006**). "Mapping seabed variability: Rapid surveying of coastal regions," J. Acoust. Soc. Am. **119**, 1373–1387.

Kamminga, S. D., Ellis, D. D., and Gerstoft, P. (**1993**). "Extraction of both bottom backscattering strength and reflection loss by inversion of reverberation measurements," J. Acoust. Soc. Am. **94**, 1844.

LePage, K. and Harrison, C. H. (**2003**). "Bistatic reverberation benchmarking exercise: BiStaR versus analytic formulas," J. Acoust. Soc. Am. **113**, 2333.

Makris, N. C. (**1993**). "Imaging ocean-basin reverberation via inversion," J. Acoust. Soc. Am. **94**, 983–993.

Muller, S. H. E., Ainslie, M. A., Boek, W., and Simons, D. G. (**2002**). "Inversion for bottom parameters using low frequency reverberation data," edited by A. Stepnowski, R. Salamon and A. Partyka, in *Proceedings of the 6th European Conference on Underwater Acoustics (ECUA)*, Poland, 24–27 June 2002, pp. 147–152.

Nielsen, P. L. (**2006**). "Simultaneous geoacoustic inversion of propagation and reverberation data," edited by S. M. Jesus and O. C. Rodriguez, in *Proceedings of the 8th European Conference on Underwater Acoustics (ECUA)*, Portugal, 12–15 June 2006, pp. 541–546.

Preston, J. R. (**2000**). "Inversion of reverberation data for rapid environmental assessment," J. Acoust. Soc. Am. **107**, 2772.

Prior, M. K. and Harrison, C. H. (**2004**). "Estimation of seabed reflection loss properties from direct blast pulse shape," J. Acoust. Soc. Am. **116**, 1341–1344.

Rogers, A., Muncill, G., and Neumann, P. (**1998**). "Inversion of bistatic reverberation and scattering measurements for seabottom properties," J. Acoust. Soc. Am. **103**, 3096.

Weston, D. E. (**1971**). "Intensity-range relations in oceanographic acoustics," J. Sound Vib. **18**, 271–287.

Weston, D. E. (**1976**). "Propagation in water with uniform sound velocity but variable-depth lossy bottom," J. Sound Vib. **47**, 473–483.

Weston, D. E. (**1980**). "Acoustic flux formulas for range-dependent ocean ducts," J. Acoust. Soc. Am. **68**, 269–281.

Zhou, J., Guan, D., Shang, E., and Luo, E. (**1982**). "Long-range reverberation and bottom scattering strength in shallow water," Chinese J. Acoust. **1**, 54–63.

Zhou, J., Zhang, X., Luo, E., and Wang, D. (**1993**). "Vertical coherence of long-range reverberation in a Pekeris shallow-water waveguide," J. Acoust. Soc. Am. **93**, 2286.

Zhou, J. and Zhang, X. (**2003**). "Seabottom geoacoustic inversion from reverberation vertical coherence in shallow water," J. Acoust. Soc. Am. **113**, 2204.

Document Data Sheet

Security Classification		Project No.
Document Serial No.	**Date of Issue**	**Total Pages**
NURC-PR-2007-001	February 2007	pp. 14

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Title

Separability of seabed reflection and scattering properties in reverberation inversion

Abstract

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Keywords

Issuing Organization

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(New York) APO AE 09613-5000]

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