A Constrained Least Squares Approach for 2D PBR Localization

Augusto Aubry[a], Vincenzo Carotenuto[b], Antonio De Maio[a], and Luca Pallotta[c]

[a]Università di Napoli "Federico II", Department of Electrical Engineering and Technology Information (DIETI), via Claudio 21, I-80125 Napoli, Italy;
[b]CNIT udr Università "Federico II", via Claudio 21, I-80125 Napoli, Italy;
[c]Università "Roma Tre" (previously with CNIT), Engineering Department, via Vito Volterra 62, I-00146 Roma, Italy.

ABSTRACT

A new algorithm for Passive Bistatic Radar (PBR) localization exploiting multiple illuminators of opportunity is proposed. To capitalize a-priori information on the receiving antenna beampattern extent, specific constraints are forced to the target localization process. At the estimator design stage the problem is formulated according to the constrained Least Squares (LS) framework. The results highlight that interesting improvements with respect to some counterparts can be achieved.

Keywords: Passive Bistatic Radar (PBR), Multiple Transmitter of Opportunity, Elliptic Localization, Range Measurements

1. INTRODUCTION

Passive Bistatic Radar (PBR) is widely recognized among the most promising and effective passive sensing technologies.[1,2] PBR based localization is accomplished using bistatic target range measurements obtained collecting the target echoes resulting from the signals transmitted by one/multiple illuminators of opportunity.[2,3] To this end, PBR receivers are equipped with two receiving channels per transmitter of oppurtunity: one is used to acquire the direct path signal from the selected emitter, the other to gather the induced echoes. Each pair of measurements allows to localize the target over an ellipse and the intersection of multiple ellipsoids due to different illuminator-passive receiver pairs paves the way for target positioning.[2,3]

This paper proposes an innovative approach for elliptic location with reference to a 2D PBR receiver exploiting multiple transmitters of opportunity. At the design stage, it is assumed that the receiving antenna has a 2D directional beampattern and specific constraints to account for its main-beam size are enforced to the target localization process. Therefore, positioning is formulated as a constrained Least Squares (LS) estimation whose optimal solution provides the Cartesian coordinates of the target. The resulting non-convex optimization problem can be efficiently solved resorting to the theory of Generalized Trust Region Subproblems (GTRSs),[4] already exploited for the case of hyperbolic location.[5] At the analysis stage, the performance of the proposed algorithm is assessed in comparison with the unconstrained LS positioning solution, the Two-Step Estimation (TSE) procedure[6] which estimates the target position via an iterative algorithm, and the CRLB on the target Cartesian coordinates that provides a benchmark on the achievable estimation accuracy.

1.1 Notation

We adopt the notation of using boldface for vectors $\boldsymbol{a}$ (lower case), and matrices $\boldsymbol{A}$ (upper case). The n-th element of $\boldsymbol{a}$ and the (m, l)-th entry of $\boldsymbol{A}$ are respectively denoted by a_n and $\boldsymbol{A}_{m,n}$. The symbols $(\cdot)^T$ indicates the transpose operator, while $\mathrm{tr}\,(\cdot)$ is the trace of its matrix argument. $\boldsymbol{A}^\dagger$ represents the Moore-Penrose inverse of the matrix $\boldsymbol{A}$. $\boldsymbol{I}$ and $\mathbf{0}$ denote respectively the identity matrix and the matrix with zero entries (their size is determined from the context). $\mathbb{R}^N$, $\mathbb{R}^{N,M}$, and $\mathbb{S}^N$ are respectively the sets of N-dimensional vectors of real

Further author information: (Send correspondence to A. De Maio)
A. De Maio: E-mail: ademaio@unina.it, Telephone: +39 081 76 83101

numbers, of $N \times M$ real matrices, and of $N \times N$ symmetric matrices. $\mathbf{diag}\,(\boldsymbol{a})$ indicates the diagonal matrix whose i-th diagonal element is the i-th entry of $\boldsymbol{a}$. The curled inequality symbol $\succeq$ (and its strict form $\succ$) is used to indicate generalized matrix inequality: for any $\boldsymbol{A} \in \mathbb{S}^N$, $\boldsymbol{A} \succ \boldsymbol{0}$ means that $\boldsymbol{A}$ is a positive definite matrix. $\lambda_1(\boldsymbol{X}), \lambda_2(\boldsymbol{X}), \ldots, \lambda_N(\boldsymbol{X})$, with $\lambda_1(\boldsymbol{X}) \geq \lambda_2(\boldsymbol{X}) \geq \ldots \geq \lambda_N(\boldsymbol{X})$, denote the eigenvalues of $\boldsymbol{X} \in \mathbb{S}^N$, arranged in decreasing order. Furthermore, given $\boldsymbol{B} \succ \boldsymbol{0}$ and $\boldsymbol{A} \in \mathbb{S}^N$, the generalized eigenvalues of the matrix pair $(\boldsymbol{A}, \boldsymbol{B})$ are given by $\lambda_i(\boldsymbol{A}, \boldsymbol{B}) = \lambda_i(\boldsymbol{B}^{-1/2}\boldsymbol{A}\boldsymbol{B}^{-1/2})$, $i = 1, \ldots, N$. The Euclidean norm of the vector $\boldsymbol{x}$ is denoted by $\|\boldsymbol{x}\|$. The letter i often serves as index. For any optimization Problem $\mathcal{P}$, $v(\mathcal{P})$ represents its optimal value.

2. SYSTEM MODEL

Consider a 2D passive bistatic radar exploiting multiple transmitters of opportunity. Then, denote by:

- $(x_p, y_p) \in \mathbb{R}^2$ the target position;
- $(x_0, y_0) \in \mathbb{R}^2$ the receiver position (without loss of generality, it is assumed to coincide with the reference system origin, i.e., $(x_0, y_0) = (0, 0)$);
- $(x_{t_i}, y_{t_i}) \in \mathbb{R}^2$ the position of the i-th transmitter of opportunity, $i = 1, \ldots, N$;
- $L_i = \sqrt{(x_{t_i} - x_0)^2 + (y_{t_i} - y_0)^2}$ the distance between the i-th transmitter and the receiver.
- $\boldsymbol{p} = [x_p, y_p]^T$ the target position vector.

At the receiver side, after the classic PBR cross-correlation based processing, the following N delay/range measurements are available

$$\tau_i = \tilde{\tau}_i + n_i, \; i = 1, \ldots, N, \tag{1}$$

where, for $i = 1, \ldots, N$,

$$\tilde{\tau}_i = \frac{1}{c}\left(\|\boldsymbol{p}\| + \sqrt{(x_p - x_{t_i})^2 + (y_p - y_{t_i})^2} - L_i\right), \tag{2}$$

with c the speed of light and $n_1, \ldots, n_N$ statistically independent zero-mean (usually assumed Gaussian distributed, even if this assumption is not mandatory for the present paper) random variables with variance $\sigma_1^2, \ldots, \sigma_N^2$. In particular,

$$\sigma_i = \frac{\sqrt{2}}{B_i\sqrt{\text{SNR}_i}}, \; i = 1, \ldots, N, \tag{3}$$

where B_i is the frequency bandwidth of the i-th signal of opportunity and SNR_i is the Signal to Noise Ratio (SNR) of the i-th bistatic pair (i.e., receiver/i-th transmitter of opportunity) evaluated according to the bistatic radar range equation.[7,8]

Now, elaborating on (2), it is possible to get an equivalent form which is fundamental for the proposed estimation algorithm (referred to in the following as Angular Constrained Least Square (ACLS)). To this end, denoting by

$$b_i = \tilde{\tau}_i c + L_i \quad\; i = 1, \ldots, N\,, \tag{4}$$

equation (2) can be recast as

$$b_i - \sqrt{x_p^2 + y_p^2} = \sqrt{(x_p - x_{t_i})^2 + (y_p - y_{t_i})^2}\,, i = 1, \ldots, N, \tag{5}$$

or equivalently as

$$\begin{cases} b_i^2 + r^2 - 2b_i r = r^2 + r_i^2 - 2x_{t_i}x_p - 2y_{t_i}y_p, \\ r \leq b_i & i = 1, \ldots, N, \\ r = \sqrt{x_p^2 + y_p^2} \end{cases} \tag{6}$$

with $r_i = \sqrt{x_{t_i}^2 + y_{t_i}^2}$, $i = 1, \ldots, N$. All the relationships described in (6) can be also framed in a more compact matrix form

$$\begin{cases} \boldsymbol{A}\bar{\boldsymbol{p}} - \boldsymbol{g} = \boldsymbol{0} \\ \bar{\boldsymbol{p}}^T \boldsymbol{B} \bar{\boldsymbol{p}} = 0 \\ \bar{p}_3 \leq b_i, \; i = 1, \ldots, N \end{cases} \tag{7}$$

where

- $\bar{\boldsymbol{p}} = [\boldsymbol{p}^T, r]^T \in \mathbb{R}^3$;
- $\boldsymbol{A}^T = [\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_N] \in \mathbb{R}^{3,N}$, with $\boldsymbol{a}_i = [-2x_{t_i}, -2y_{t_i}, 2b_i]^T \in \mathbb{R}^3$, $i = 1, \ldots, N$
- $\boldsymbol{g} = [b_1^2 - r_1^2, b_2^2 - r_2^2, \ldots, b_N^2 - r_N^2]^T \in \mathbb{R}^N$;
- $\boldsymbol{B} = \textbf{diag}\,\{[1, 1, -1]\} \in \mathbb{R}^{3,3}$.

2.1 Receiver Beampattern Constraint

To perform the search process, conventional 2D PBR systems employ a scanning antenna (in the azimuth domain) characterized by a specific beampattern and in particular a given mainlobe width. In this subsection, some constraints able to capitalize such a-priori information are formalized with the goal of improving target localization reliability. To this end, let us denote by:

- $\bar{\theta}$ the receiving (half-side) antenna beamwidth (with respect to the boresight), with $0 \leq \bar{\theta} < \pi/2$;
- $\theta \in \,]-\pi, \pi[$ the squint angle of the antenna boresight with respect to the x-axis;
- For $(x, y) \neq (0, 0)$, $\theta_p = \text{atan2}(y, x)$ the target angle of arrival, where $\text{atan2}(\cdot)$ is the four-quadrants inverse tangent.

Suppose, now, that the target is within the receiving antenna main-beam, i.e., that

$$\theta - \bar{\theta} \leq \theta_p \leq \theta + \bar{\theta} \tag{8}$$

or equivalently

$$-\bar{\theta} \leq \theta_p - \theta \leq \bar{\theta}. \tag{9}$$

Otherwise stated, it is assumed that the detector performs correctly its task, i.e., the target triggering the detection resides in the antenna main-beam.

Equation (9) coupled with the assumption $0 \leq \bar{\theta} < \frac{\pi}{2}$ can be equivalently rewritten as

$$-\tan \bar{\theta} \leq \tan\left(\theta_p - \theta\right) \leq \tan \bar{\theta} \tag{10}$$

and

$$|\theta_p - \theta| < \frac{\pi}{2}. \tag{11}$$

Now, the following relationships on the tangent function hold

$$\tan\left(\theta_p - \theta\right) = \frac{\sin(\theta_p - \theta)}{\cos(\theta_p - \theta)} = \frac{\sin(\theta_p)\cos(\theta) - \cos(\theta_p)\sin(\theta)}{\cos(\theta_p)\cos(\theta) + \sin(\theta_p)\sin(\theta)} = \frac{y_p \cos\theta - x_p \sin\theta}{x_p \cos\theta + y_p \sin\theta}, \tag{12}$$

where the last equality stems from $\sin\theta_p = y_p/\sqrt{x_p^2 + y_p^2}$ and $\cos\theta_p = x_p/\sqrt{x_p^2 + y_p^2}$.

Exploiting (12), (10) can be written in a more useful form i.e. as

$$-\tan\bar{\theta} \leq \frac{y_p\cos\theta - x_p\sin\theta}{x_p\cos\theta + y_p\sin\theta} \leq \tan\bar{\theta}. \tag{13}$$

Let us now manipulate the previous inequalities introducing a new reference system, say (x_1, y_1), obtained rotating the actual one (i.e., the (x, y)-coordinates system) such that the x_1-axis is aligned with the receiving antenna boresight. Precisely, denoting by $\bar{\boldsymbol{R}}(\theta)$ the rotation matrix of an angle θ clockwise, i.e.,

$$\bar{\boldsymbol{R}}(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}, \tag{14}$$

the new (x_1, y_1)-coordinates system is related to the previous (x, y)-coordinates system by means of the following transformation

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \bar{\boldsymbol{R}}(\theta)\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x\cos\theta + y\sin\theta \\ -x\sin\theta + y\cos\theta \end{bmatrix}. \tag{15}$$

Consequently, (13) becomes

$$-\tan\bar{\theta} \leq \frac{y_{1p}}{x_{1p}} \leq \tan\bar{\theta}. \tag{16}$$

From the relationship

$$|\text{atan2}\,(y_{1p}, x_{1p})| = |\text{atan2}\,(y, x) - \theta| < \frac{\pi}{2}, \tag{17}$$

it directly follows that $x_{1p} > 0$. Hence, the additional constraints to consider at the estimator design stage as induced by the antenna beamwidth are given by

$$\begin{cases} -x_{1p}\tan\bar{\theta} \leq y_{1p} \leq x_{1p}\tan\bar{\theta} \\ x_{1p} > 0 \end{cases}. \tag{18}$$

Finally, to take in account also the point with coordinates $(x_{1p}, y_{1p}) = (0, 0)$ and letting $\gamma = \tan\bar{\theta}$, the constraints in (18) can be expressed as

$$\begin{cases} -x_{1p}\gamma \leq y_{1p} \leq x_{1p}\gamma \\ x_{1p} \geq 0 \\ \begin{bmatrix} x_{1p} \\ y_{1p} \end{bmatrix} = \bar{\boldsymbol{R}}(\theta)\begin{bmatrix} x_p \\ y_p \end{bmatrix} \end{cases}. \tag{19}$$

3. PROBLEM FORMULATION

Starting from (7), it can be observed that the first equation holds only approximately in practical situations due to measurement errors (1). A viable mean to overcome this shortcoming is to resort to the constrained LS framework and formalize the estimation problem as

$$\begin{cases} \min\limits_{\bar{\boldsymbol{p}}} & \|\boldsymbol{A}\bar{\boldsymbol{p}} - \boldsymbol{g}\|^2 \\ \text{s.t.} & \bar{\boldsymbol{p}}^T\boldsymbol{B}\bar{\boldsymbol{p}} = 0 \\ & 0 \leq \bar{p}_3 \leq b \end{cases} \tag{20}$$

with $b = \min\limits_{i=1,\ldots,N} b_i$. Notice that, with a slight abuse of notation, the model parameters in (20), i.e., $\boldsymbol{A}$, $\boldsymbol{g}$, and b, are computed exploiting the actual measurements τ_i, $i = 1, \ldots, N$, in place of $\tilde{\tau}_i$, $i = 1, \ldots, N$.

In the current form the considered estimation problem (20) does not account for the receive antenna main-beam constraints introduced in 2.1. Thus, to proceed further, let us introduce the target position vector in the rotated (x_1, y_1)-coordinates reference system, indicated in the following as $\tilde{\boldsymbol{p}}$. Precisely, the position vector in the rotated coordinate system is given by $\tilde{\boldsymbol{p}} = \boldsymbol{U}^T \bar{\boldsymbol{p}}$, where

$$\boldsymbol{U} = \begin{bmatrix} \bar{\boldsymbol{R}}(\theta)^T & 0 \\ 0 & 1 \end{bmatrix} \tag{21}$$

with $\boldsymbol{U}^T\boldsymbol{U} = \boldsymbol{I}$. As a consequence, elliptic localization problem with the addition of the beampattern constraints (19), is formulated as

$$\mathcal{P} \begin{cases} \min_{\tilde{\boldsymbol{p}}} \left\| \tilde{\boldsymbol{A}}\tilde{\boldsymbol{p}} - \boldsymbol{g} \right\|^2 \\ \text{s.t. } \tilde{\boldsymbol{p}}^T \boldsymbol{B} \tilde{\boldsymbol{p}} = 0 & \text{(22a)} \\ \quad 0 \le \tilde{p}_3 \le b & \text{(22b)} \\ \quad -\tilde{p}_1\gamma \le \tilde{p}_2 \le \tilde{p}_1\gamma & \text{(22c)} \\ \quad \tilde{p}_1 \ge 0 & \text{(22d)} \end{cases}$$

where $\tilde{\boldsymbol{A}} = \boldsymbol{A}\boldsymbol{U}$ and $\boldsymbol{U}^T\boldsymbol{B}\boldsymbol{U} = \boldsymbol{B}$.

Problem $\mathcal{P}$ is a non-convex optimization problem apparently difficult to solve. However, leveraging the special structure of the objective function/constraints and resorting to the theory of GTRSs,[4] a closed-form optimal solution to $\mathcal{P}$ is now derived. A first step toward this goal is provided by the following lemma

LEMMA 3.1. *Any feasible point $\boldsymbol{x} \neq (0,0,0)$ is regular for the optimization Problem $\mathcal{P}$.*

The following proposition establishes a procedure leading to a closed-form global optimal solution of the constrained estimation problem $\mathcal{P}$. This represents the main technical contribution of the present paper from an optimization theory point of view.

PROPOSITION 3.2. *An optimal solution to $\mathcal{P}$ belongs to the following finite set of feasible points (whose cardinality is at most thirteen):*

1. $\bar{\boldsymbol{x}}_0^\star = \boldsymbol{0}$.
2. $\bar{\boldsymbol{x}}^\star(\eta_h) = \left(\tilde{\boldsymbol{A}}^T\tilde{\boldsymbol{A}} + \eta_h \boldsymbol{B}\right)^{-1} \tilde{\boldsymbol{A}}^T \boldsymbol{g}$, *where η_h, $h = 1, \ldots, N_r$ $(N_r \le 4)$, are the roots of the fourth order equation*
$$\bar{\boldsymbol{x}}^\star(\eta)^T \boldsymbol{B} \bar{\boldsymbol{x}}^\star(\eta) = 0 \tag{23}$$
with
$$\eta \in \left(-\frac{1}{\lambda_2\left(\boldsymbol{B}, \tilde{\boldsymbol{A}}^T\tilde{\boldsymbol{A}}\right)}, +\infty\right) - \left\{-\frac{1}{\lambda_1\left(\boldsymbol{B}, \tilde{\boldsymbol{A}}^T\tilde{\boldsymbol{A}}\right)}, -\frac{1}{\lambda_3\left(\boldsymbol{B}, \tilde{\boldsymbol{A}}^T\tilde{\boldsymbol{A}}\right)}\right\} \tag{24}$$
such that
$$\begin{cases} 0 < \bar{x}_3^\star(\eta_h) < b \\ -\gamma\bar{x}_1^\star(\eta_h) \le \bar{x}_2^\star(\eta_h) \le \gamma\bar{x}_1^\star(\eta_h) \\ \bar{x}_1^\star(\eta_h) > 0 \end{cases} . \tag{25}$$
3. $\bar{\boldsymbol{x}}^\star(\beta_h) = [\tilde{\boldsymbol{q}}^\star(\beta_h)^T, b]^T$ *with*
$$\tilde{\boldsymbol{q}}^\star(\beta_h) = \left(\tilde{\boldsymbol{A}}_1^T\tilde{\boldsymbol{A}}_1 + \beta_h \boldsymbol{I}\right)^{-1} \tilde{\boldsymbol{A}}_1^T (\boldsymbol{g} - \tilde{\boldsymbol{a}}_3 b),$$
where $\tilde{\boldsymbol{A}} = \left[\tilde{\boldsymbol{A}}_1, \tilde{\boldsymbol{a}}_3\right]$ and β_h, $h = 1, \ldots, N_{r_1}$ $(N_{r_1} \le 4)$ are the roots of the fourth order equation
$$\tilde{\boldsymbol{q}}^\star(\beta)^T \tilde{\boldsymbol{q}}^\star(\beta) = b \tag{26}$$

such that

$$\begin{cases} \beta_h \geq -\lambda_{max}\left(\tilde{\boldsymbol{A}}_1^T\tilde{\boldsymbol{A}}_1\right) - \left\{-\lambda_{min}\left(\tilde{\boldsymbol{A}}_1^T\tilde{\boldsymbol{A}}_1\right)\right\} \\ -\gamma\tilde{q}_2^\star(\beta_h) < \tilde{q}_1^\star(\beta_h) < \gamma\tilde{q}_2^\star(\beta_h) \\ \tilde{q}_1^\star(\beta_h) > 0 \end{cases}. \quad (27)$$

4. $\bar{\boldsymbol{x}}_{4_i}^\star = \left[b/\sqrt{1+\gamma^2}, (-1)^{i+1}\gamma b/\sqrt{1+\gamma^2}, b\right]^T$, *for* $i = 1,2$.

5. $\bar{\boldsymbol{x}}_{5_i}^\star = \alpha_\pm^\star\left[1, (-1)^{i+1}\gamma, \sqrt{1+\gamma^2}\right]^T$, $i = 1,2$, *with*

$$\alpha_\pm^\star = \max\left(0, \frac{v_{\pm,1}g_1 + v_{\pm,2}g_2 + v_{\pm,3}g_3}{v_{\pm,1}^2 + v_{\pm,2}^2 + v_{\pm,3}^2}\right).$$

and $\boldsymbol{v}_\pm = \tilde{\boldsymbol{A}}\left[1, \pm\gamma, \sqrt{1+\gamma^2}\right]^T$.

4. PERFORMANCE ANALYSIS

This section is devoted to the performance assessment of the proposed target localization algorithm for a PBR exploiting multiple transmitters of opportunity. As case study, a localization scenario comprising $N = 3$ omni-directional broadcast transmitters of opportunity is analyzed. As to the specific geometric configuration, the transmitters are located at the vertices of an equilateral triangle whose barycenter is the position of the receiver, that coincides with the origin of the reference system. Precisely, the distance of each transmitter of opportunity from the receiver is $L_i = 10$ km, $i = 1,2,3$. The considered setup is graphically illustrated in Figure 1.

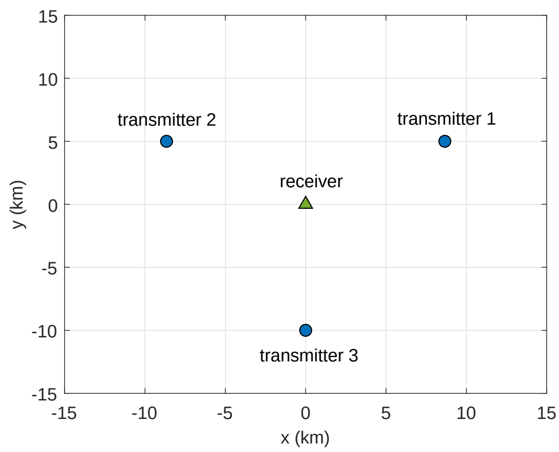


Figure 1. Geometric configuration composed of a receiver at the reference system origin and $N = 3$ transmitters of opportunity located at the vertices of an equilateral triangle ($L_i = 10$ km, $i = 1,2,3$).

Modelling the measurement errors according to (3), the SNR of the $N = 3$ bistatic pairs (receiver-transmitter of opportunity) is set as[7]

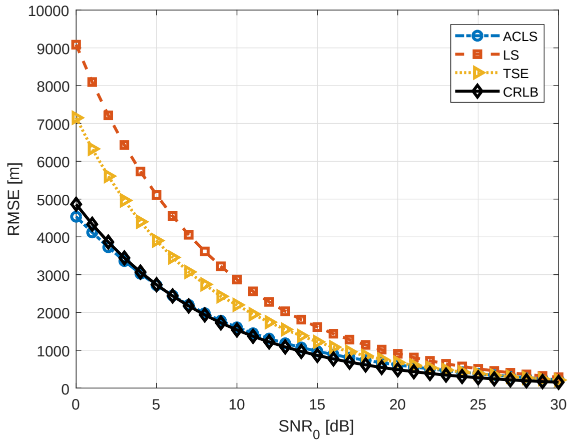
$$\mathrm{SNR}_i = \mathrm{SNR}_0 \frac{\|\boldsymbol{q}_0\|^2}{\|\boldsymbol{p}\|^2}\frac{\|\boldsymbol{q}_0 - \boldsymbol{t}_1\|^2}{\|\boldsymbol{p} - \boldsymbol{t}_i\|^2}, \quad i = 1,2,3, \quad (28)$$

where $\boldsymbol{t}_i = [x_{t_i}, y_{t_i}]^T$, $i = 1,2,3$, are the locations of the transmitters of opportunity, $\boldsymbol{p}$ is the true target position, and SNR_0 is a reference SNR value computed via the bistatic radar range equation[7,8] related to a reference point $\boldsymbol{q}_0 = [\bar{x}, \bar{y}]^T$ and considering the transmitter located at $\boldsymbol{t}_1$ as radiation source.

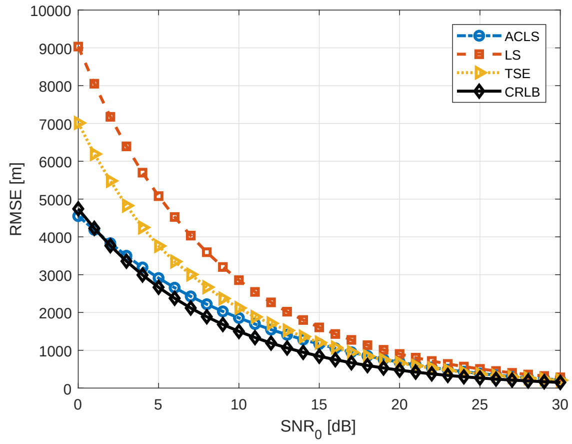
The performance of the devised localization algorithm is assessed considering as figure of merit the Root Mean Square Error (RMSE) of the target position estimate. To this end, due to the lack of a closed-form expression for the RMSE, Monte Carlo simulation method is employed, performing 1000 independent runs. The analyses are conducted also in comparison with the LS and TSE[6] algorithms, and with the CRLB benchmark, i.e. $\sqrt{\mathrm{tr}\left(\mathrm{FIM}^{-1}\right)}$ with FIM the Fisher Information Matrix.

The following study considers a target located at $(x_p, y_p) = (r\cos\theta_p, r\sin\theta_p)$ with $r = 40$ km and different values of θ_p, $\theta_p = 0, 7, 9$ deg. Moreover, the receiving antenna is assumed steered at $\theta = 0$ deg with a main-beam width of $\bar{\theta} = 10$ deg.

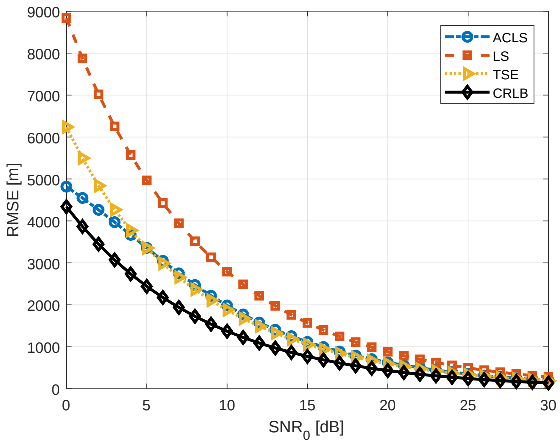
The obtained results are reported in Figure 2, where the RMSE is plotted versus SNR_0 for $B_i = 150$ kHz (representative of FM radio stations), $i = 1, 2, 3$. Therein, SNR_0 coincides with the actual SNR, regardless of the target position. Indeed, it is assumed that the reference point $\boldsymbol{q}_0$ involved in the SNR computation coincides with the actual target position $\boldsymbol{p}$, therefore $\boldsymbol{q}_0$ changes among the subplots.



(a) $\theta_p = 9°$.



(b) $\theta_p = 7°$.



(c) $\theta_p = 0°$.

Figure 2. RMSE versus SNR_0 assuming $\boldsymbol{p} = \boldsymbol{q}_0$ considering $B_i = 150$ kHz, $i = 1, 2, 3$.

The curves show that for low SNR values, the ACLS provides better performance than the unconstrained LS and TSE counterparts, clearly revealing the effectiveness of the new procedure to exploit a-priori information on the beampattern main-beam size. Specifically, the RMSE values achieved by ACLS are close to the CRLB. Interestingly, as the SNR increases, all the considered positioning algorithms, i.e., ACLS, unconstrained LS, and

TSE, improve their location estimation performance providing RMSE values converging to the CRLB benchmark. Finally, it is worth observing that the closer the target position to the beampattern boundary the better the performance of the ACLS method as compared with the counterparts. This is not surprising since the a-priori information on the beampattern extent becomes more and more valuable when the target lies in proximity of the boundary region.

5. CONCLUSIONS

In this paper an innovative approach for the elliptic location has been proposed with reference to a PBR receiver that exploits multiple transmitters of opportunity. To this end, at the design stage, some constraints have been enforced to the target localization process in order to exploit a-priori information on the receiving antenna main-beam size. Therefore, the problem has been formulated as a constrained LS estimation whose optimal solution provides the Cartesian coordinates of the target. To handle the resulting non-convex optimization problem, an efficient solution technique able to provide a global optimal point has been provided.

The conducted analyses have demonstrated the effectiveness of the proposed algorithm also in comparison with other counterparts available in the open literature especially for low SNRs. Precisely, for the considered study cases, the curves have highlighted that the RMSE values attained by the novel technique are usually lower than those of the competitors, therefore validating the benefits provided by the additional constraints on the receiving beampattern. As a matter of fact, the obtained results have shown values very close to those of theoretical CRLB.

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