Synthesis of Constant Modulus Radar Signals in Spectrally Crowded Scenarios

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ABSTRACT

This paper deals with the synthesis of waveforms optimizing radar performance while satisfying multiple spectral compatibility constraints. Precisely, on each shared bandwidth a precise and specific control on the injected interference energy is forced. Furthermore, constant modulus waveforms are considered at the design stage to comply with today amplifiers technology. To tackle the resulting NP-hard optimization problem, an iterative procedure based on the coordinate descent method is introduced. Hence, some case studies are reported to highlight the effectiveness of the technique.

Keywords: Waveform design, Spectral compatibility, Optimization theory

1. INTRODUCTION

Recent years have witnessed a growing interest in the Radio Frequency (RF) congestion problem being RF spectrum a commodity necessary for an ever-growing number of services and systems.[1] In particular, wider and wider amount of bandwidths are required by advanced wireless communication and remote sensing systems to accomplish their demanding tasks, e.g., high data-rates and reliable surveillance. As a consequence, a serious challenge is posed on the overall spectral usability by non-cooperative systems[2] and spectrum compatibility issue has attracted the interest of many scientists and engineers, currently becoming one amongst the hot topics in both regulation and research field. In particular, many papers in the open literature have dealt with the problem of designing radar signals with a suitable frequency allocation so as to induce acceptable interference levels on frequency-overlaid systems (see[3–10] and references therein).

In this paper, a new technique for constant modulus waveform synthesis specifically designed to face spectrally dense environments is proposed, where the Interference plus Noise Ratio (SINR) serves as the preferred performance metric. A local control on the interference energy radiated on each shared/reserved frequency bandwidth is performed. Along with a requirement on the maximum transmitted energy, a similarity constraint is enforced on the probing sequence in order to control significant waveform characteristics, e.g., Peak Sidelobe Level (PSL) and ISL. The optimization process is restricted to constant modulus waveforms for compatibility with current amplifier technology and is unprecedented in terms of synthesizing constant modulus codes subject to multiple spectral compatibility constraints. The resulting optimization problem belongs to the class of NP-hard problems and a new iterative algorithm based on the Coordinate Descent (CD) method is developed to synthesize optimized codes. At the analysis stage, some interesting case studies are illustrated to assess the capability of the devised algorithm to improve radar detection performance while guaranteeing spectral compatibility.

The paper is organized as follows. Section 2 introduces the system model and defines the waveform design problem. In Section 3, an efficient algorithm to handle the formulated optimization problem is devised. Section 4 is devoted to the performance analysis. Finally, Section 5 concludes the paper.

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Notation

We adopt the notation of using boldface for vectors $\boldsymbol{a}$ (lower case), and matrices $\boldsymbol{A}$ (upper case). The n-th element of $\boldsymbol{a}$ and the (n,m)-th entry of $\boldsymbol{A}$ are denoted by a_n and $\boldsymbol{A}_{n,m}$, respectively. The transpose and the conjugate transpose operators are denoted by the symbols $(\cdot)^T$ and $(\cdot)^\dagger$ respectively. $\boldsymbol{I}$ and $\boldsymbol{0}$ denote respectively the identity matrix and the matrix with zero entries (their size is determined from the context). $\mathbb{R}^N$, $\mathbb{C}^N$, and $\mathbb{H}^N$ are respectively the sets of N-dimensional vectors of real numbers, of N-dimensional vectors of complex numbers, and of $N \times N$ Hermitian matrices. The curled inequality symbol $\succeq$ (and its strict form $\succ$) is used to denote generalized matrix inequality: for any $\boldsymbol{A} \in \mathbb{H}^N$, $\boldsymbol{A} \succeq \boldsymbol{0}$ means that $\boldsymbol{A}$ is a positive semi-definite matrix ($\boldsymbol{A} \succ \boldsymbol{0}$ for positive definiteness). For any complex number x, $|x|$ represents the modulus of x. Moreover, for any $\boldsymbol{x} \in \mathbb{C}^N$, $\|\boldsymbol{x}\|$ denotes the Euclidean norm and $\mathbf{diag}\,(\boldsymbol{x})$ indicates the diagonal matrix whose i-th diagonal element is the i-th entry of $\boldsymbol{x}$. Finally, $\mathbb{E}\left[\cdot\right]$ denotes statistical expectation and for any optimization problem $\mathcal{P}$, $v(\mathcal{P})$ is its optimal value.

2. SYSTEM MODEL

Let $\boldsymbol{c} = [c_1, \ldots, c_N]^T \in \mathbb{C}^N$ be the transmitted fast-time radar code vector with N the number of coded sub-pulses (code length). The waveform at the receiver end is down-converted to baseband, undergoes a sub-pulse matched filtering operation, and then is sampled. As a result, the N-dimensional column vector $\boldsymbol{v} = [v_1, \ldots, v_N]^T \in \mathbb{C}^N$ of the fast-time observations from the range-azimuth cell under test can be expressed as

$$\boldsymbol{v} = \alpha_T \boldsymbol{c} + \boldsymbol{n},$$

with α_T a complex parameter accounting for channel propagation and backscattering effects from the target within the range-azimuth bin of interest and $\boldsymbol{n}$ the N-dimensional column vector containing the filtered disturbance signal samples; $\boldsymbol{n}$ is modeled as a complex, zero-mean, circularly symmetric Gaussian random vector with covariance matrix $\mathbb{E}[\boldsymbol{n}\boldsymbol{n}^\dagger] = \boldsymbol{M}$.

As to the licensed emitters coexisting with the radar of interest, it is supposed that each of them is operating over a frequency band $\Omega_k = \left[f_1^k, f_2^k\right]$, $k = 1, \ldots, K$, where f_1^k and f_2^k denote the lower and upper normalized frequencies for the k-th system, respectively. To ensure spectral compatibility the radar has to shape properly its transmit waveform to manage the amount of interfering energy produced on the shared frequency bandwidths given by[7]

$$\int_{f_1^k}^{f_2^k} S_c\left(f\right) df \quad = \quad \boldsymbol{c}^\dagger \boldsymbol{R}_I^k \boldsymbol{c}, \tag{1}$$

where $S_c\left(f\right) = \left|\sum_{n=0}^{N-1} \boldsymbol{c}_n e^{-j2\pi f n}\right|^2$ is the Energy Spectral Density (ESD) of the fast-time code $\boldsymbol{c}$ and for $(m,l) \in \{1, \ldots, N\}^2$

$$\boldsymbol{R}_I^k(m,l) = (f_2^k - f_1^k) e^{j\pi(f_2^k + f_1^k)(m-l)} \mathrm{sinc}\left(\pi(f_2^k - f_1^k)(m-l)\right)$$

Thus, denoting by E_I^k, $k = 1, \ldots, K$, the acceptable level of disturbance on the k-th bandwidth, that is related to the quality of service required by the k-th telecommunication networks, the transmitted waveform has to comply with the constraints

$$\boldsymbol{c}^\dagger \boldsymbol{R}_I^k \boldsymbol{c} \leq E_I^k, \quad k = 1, \ldots, K. \tag{2}$$

Remarkably, unlike several approaches proposed in the open literature, e.g.,[11] where a weighted sum of the produced interference power is usually controlled, (2) provides a detailed control of the interference energy produced on each shared frequency bandwidth. Finally, the radar system may resort to a Radio Environmental Map (REM)[12] as well as suitable spectrum sensing modules[13] to get cognizance of the licensed emitters (e.g. their spatial location and bandwidth occupation).

2.1 Code Design Optimization Problem

In this subsection, a radar waveform design framework is introduced aimed at improving target detection probability while controlling both the amount of interfering energy produced in each shared bands and some hallmarks of the transmitted waveform. To this end, the SINR, defined as

$$\mathrm{SINR}(\boldsymbol{c}) = |\alpha_T|^2 \boldsymbol{c}^\dagger \boldsymbol{R}\boldsymbol{c} \tag{3}$$

is considered as figure of merit, where $\boldsymbol{R} = \boldsymbol{M}^{-1}$ is the inverse of the interference covariance matrix. To control some attributes of the transmitted waveform, other than an energy requirement, namely $\|\boldsymbol{c}\|^2 \leq 1$, a similarity constraint is imposed on the probing signature, i.e., $\left\| \frac{\boldsymbol{c}}{\|\boldsymbol{c}\|} - \boldsymbol{c}_0 \right\|_\infty \leq \frac{\epsilon}{\sqrt{N}}$, where the parameter $0 \leq \epsilon \leq 2$ rules the size of the trust hypervolume, and $\boldsymbol{c}_0 = [c_{0,1}, \ldots, c_{0,N}]^T \in \mathbb{C}^N$ is a specific phase-only reference code, with $\|\boldsymbol{c}_0\|^2 = 1$. Indeed, an unconstrained optimization can lead to signals with high PSL/ISL values and more generally with an undesired ambiguity function behavior. These drawbacks can be partially circumvented forcing the solution to be similar to a known code $\boldsymbol{c}_0$ that shares some advantageous properties. Finally, to assure compatibility with current amplifier technology, the optimization process is restricted to constant modulus waveforms.

Summarizing, leveraging on the aforementioned guidelines, the waveform design problem of interest can be formulated as the following non-convex optimization problem

$$\mathcal{P} \begin{cases} \max\limits_{\boldsymbol{c}\in\mathbb{C}^N} & \overline{\mathrm{SINR}}(\boldsymbol{c}) \\ \text{s.t.} & \boldsymbol{c}^\dagger \boldsymbol{R}_I^k \boldsymbol{c} \leq E_I^k, k = 1, \ldots, K \\ & \|\boldsymbol{c}\|^2 \leq 1 \\ & |c_i| = |c_k|, \, (i,k) \in \{1, \ldots, N\}^2 \\ & \left\| \frac{\boldsymbol{c}}{\|\boldsymbol{c}\|} - \boldsymbol{c}_0 \right\|_\infty \leq \frac{\epsilon}{\sqrt{N}} \end{cases} \tag{4}$$

where

$$\overline{\mathrm{SINR}}(\boldsymbol{c}) = \boldsymbol{c}^\dagger \boldsymbol{R}\boldsymbol{c}. \tag{5}$$

is the normalized SINR. Furthermore, re-parameterizing the optimization vector $\boldsymbol{c}$ as

$$\boldsymbol{c} = \sqrt{\frac{\rho}{N}} \left[e^{j(\psi_1 + \arg(c_{0,1}))}, \ldots, e^{j(\psi_N + \arg(c_{0,N}))} \right]^T \in \mathbb{C}^N,$$

with $\psi_i \in [-\pi, \pi]$, $i = 1, \ldots, N$, and $0 \leq \rho \leq 1$, the design problem boils down to

$$\bar{\mathcal{P}} \begin{cases} \max\limits_{\boldsymbol{x}} & \overline{\mathrm{SINR}}\,(\boldsymbol{x}) \\ \text{s.t.} & \bar{\boldsymbol{c}} = \sqrt{x_{N+1}} \left[e^{jx_1}, e^{jx_2}, \ldots, e^{jx_N} \right]^T \\ & \bar{\boldsymbol{c}}^\dagger \bar{\boldsymbol{R}}_I^k \bar{\boldsymbol{c}} \leq E_I^k, k = 1, \ldots, K \\ & 0 \leq x_{N+1} \leq 1 \\ & |x_i| \leq \delta, \, i = 1, \ldots, N \end{cases} \tag{6}$$

where $\boldsymbol{x} = [\psi_1, \psi_2, \ldots, \psi_N, \rho]^T \in \mathbb{R}^{N+1}$ is the new optimization vector (with x_i, $i = 1, \ldots, N$, the phase-code shift with respect to $c_{0,i}$, and x_{N+1} related to the actual code amplitude), $\delta = \operatorname{arcos}\left(1 - \frac{\epsilon^2}{2}\right)$,

$$\bar{\boldsymbol{R}}_I^k = \mathbf{diag}\,(\boldsymbol{c}_0^*) \boldsymbol{R}_I^k \mathbf{diag}\,(\boldsymbol{c}_0), \tag{7}$$

controls the k-th spectral constraint in terms of the new optimization variables, and, with a slight abuse of notation, $\overline{\mathrm{SINR}}\,(\boldsymbol{x})$ is the function (5) evaluated at $\boldsymbol{c} = \sqrt{\frac{X_{N+1}}{N}} \left[e^{j(x_1 + \arg(c_{0,1}))}, \ldots, e^{j(x_N + \arg(c_{0,N}))} \right]^T$.

3. RADAR CODE OPTIMIZATION

In this section, an iterative algorithm that exploits the CD maximization paradigm[14,15] is developed to get optimized solutions with some quality guarantee to the NP-hard Problem $\mathcal{P}$. The idea is to alternate the maxization of the objective (5) among the entries of $\boldsymbol{x}$, namely optimizing one variable at time while keeping fixed the others. This is tantamount to solving appropriate univariate optimization problems in a loop.[14] With reference to $\bar{\mathcal{P}}$, at each iteration a specific entry of $\boldsymbol{x}$ is selected as variable to optimize. This leads to the following problem at step $n+1$

$$\bar{\mathcal{P}}_{d,\boldsymbol{x}^{(n)}} \begin{cases} \max\limits_{x_d} & \widehat{\mathrm{SINR}}\left(x_d; \boldsymbol{x}_{-d}^{(n)}\right) \\ \text{s.t.} & f_k(x_d; \boldsymbol{x}_{-d}^{(n)}) \leq E_I^k, k=1,\ldots,K \\ & g(x_d) \leq 0 \end{cases} \tag{8}$$

where $\boldsymbol{x}^{(n)}$ is the optimized vector at step n, x_d is the variable to optimize at step $n+1$, and

$$\boldsymbol{x}_{-d}^{(n)} = [x_1^{(n)}, \ldots, x_{d-1}^{(n)}, x_{d+1}^{(n)}, \ldots, x_{N+1}^{(n)}]^T \in \mathbb{R}^N.$$

Moreover,

$$\widehat{\mathrm{SINR}}\left(x_d; \boldsymbol{x}_{-d}^{(n)}\right) = \overline{\mathrm{SINR}}\left(x_1^{(n)}, \ldots, x_{d-1}^{(n)}, x_d, x_{d+1}^{(n)}, \ldots, x_{N+1}^{(n)}\right),$$

is the normalized SINR function restricted to x_d only while the other parameters are those of the previous step; besides,

$$f_k(x_d; \boldsymbol{x}_{-d}^{(n)}) = \begin{cases} \left(\bar{z}_{k,d}^{(n)} + 2\Re\left\{z_{k,d}^{(n)}\, e^{-jx_d}\right\}\right) x_{N+1}^{(n)}, & d=1,\ldots,N \\ x_d\, p_k^{(n)}, & d=N+1 \end{cases}$$

is the restriction of the k-th spectral constraint, induced by (7), to x_d only keeping fixed the other variable (see[16] for the formal definition of the parameters $\bar{z}_{k,d}^{(n)}, z_{k,d}^{(n)}, p_k^{(n)}$ as well as technical details); finally,

$$g(x_d) = \begin{cases} |x_d| - \delta, & d=1,\ldots,N \\ \left|x_d - \frac{1}{2}\right| - \frac{1}{2}, & d=N+1 \end{cases}$$

Thus, denoting by $x_{d,n+1}^{\star}$ the optimal solution to $\bar{\mathcal{P}}_{d,\boldsymbol{x}^{(n)}}$, the optimized radar code at step $n+1$ is $\boldsymbol{x}^{(n+1)} = [x_1^{(n)}, \ldots, x_{d-1}^{(n)}, x_{d,n+1}^{\star}, \ldots, x_{N+1}^{(n)}]^T$. As a result, starting from a feasible solution $\boldsymbol{x}^{(0)}$ a sequence $\boldsymbol{x}^{(1)}$, $\boldsymbol{x}^{(2)}$, $\boldsymbol{x}^{(3)}$, ... of radar codes are obtained iteratively. A summary of the proposed approach can be found in **Algorithm 1**.

Remarkably, being $\boldsymbol{x}^{(n)}$ feasible to $\bar{\mathcal{P}}$ for any n,

$$\begin{aligned} \overline{\mathrm{SINR}}^{(n)} &= \widehat{\mathrm{SINR}}\left(x_d^{(n)}; \boldsymbol{x}_{-d}^{(n)}\right) \\ &\leq \widehat{\mathrm{SINR}}\left(x_{d,n+1}^{\star}; \boldsymbol{x}_{-d}^{(n)}\right) \\ &= \overline{\mathrm{SINR}}^{(n+1)} \end{aligned}$$

implying that the objective function monotonically increases along with iterations; hence, due to the fact that the objective function is bounded (from above) the convergence of the sequence of objective values holds true.

In the next subsections, efficient algorithms to tackle $\bar{\mathcal{P}}_{d,\boldsymbol{x}^{(n)}}$, $d=1,\ldots,N+1$, are developed. From an optimization theory point of view this is the main technical innovation of the paper.

Algorithm 1 Constant Modulus Code Design with Spectral Compatibility Requirements

Input: Initial feasible code $\boldsymbol{x}_0$ and the minimum required improvement $\bar{\epsilon}$;
Output: Optimal solution $\boldsymbol{x}^\star$;

1. **Initialization**.
   - Set $d := 1$ and $n := 0$ and compute the initial objective value $\overline{\text{SINR}}^{(n)} = \overline{\text{SINR}}\,(\boldsymbol{x}_0)$;
2. **Improvement**.
   - Solve $\bar{\mathcal{P}}_{d,\boldsymbol{x}^{(n)}}$ obtaining $x^\star_{d,n+1}$;
   - Set $n := n+1$,
     $\boldsymbol{x}^{(n)}=\left[x_1^{(n-1)},\ldots,x_{d-1}^{(n-1)},x^\star_{d,n},x_{d+1}^{(n-1)},\ldots,x_{N+1}^{(n-1)}\right]^T$,
     $\overline{\text{SINR}}^{(n)} = \overline{\text{SINR}}(\boldsymbol{x}^{(n)})$;
3. **Stopping Criterion**.
   - If $\text{mod}(n, N+1) = 0$ and $|\overline{\text{SINR}}^{(n)} - \overline{\text{SINR}}^{(n-1)}| < \bar{\epsilon}$, stop. Otherwise, update d, i.e., $d = \text{mod}(d-1; N+1)+1$, and go to the step 2;
4. **Output**.
   - Set $\boldsymbol{x}^\star = \boldsymbol{x}^{(n)}$.

3.1 Solution Teqcnique for $\bar{\mathcal{P}}_{d,\boldsymbol{x}^{(n)}}$

Let us start with the study of Problem $\bar{\mathcal{P}}_{d,\boldsymbol{x}^{(n)}}$ as $d = N+1$. In this case, it can shown that (8) reduces to a linear programming problem with a positive objective slope. Hence, the optimal solution is just the highest feasible value which is given by

$$x^\star_{N+1,n+1} = \min\left(\min_{k=1,\ldots,K}\left(\frac{E_I^k}{p_k^{(n)}}\right), 1\right)$$

with $p_k^{(n)}$ the parameter involved in the definition of $f_k(x_d; \boldsymbol{x}_{-d}^{(n)})$. With reference to $1 \leq d \leq N$, the following proposition paves the way for an efficient solution of $\bar{\mathcal{P}}_{d,\boldsymbol{x}^{(n)}}$.

PROPOSITION 1. *Problem $\bar{\mathcal{P}}_{d,\boldsymbol{x}^{(n)}}$, $d = 1,\ldots,N$, is equivalent to the following quadratic constrained fractional quadratic problem*

$$\hat{\mathcal{P}}_{d,\boldsymbol{x}^{(n)}} \begin{cases} \max\limits_{t} & \dfrac{a_d^{(n)}t^2 + b_d^{(n)}t - a_d^{(n)}}{1+t^2} \\ s.t. & a_{k,d}^{(n)}t^2 + b_{k,d}^{(n)}t + c_{k,d}^{(n)} \leq 0,\ k = 1,\ldots,K \\ & -\bar{\delta} \leq t \leq \bar{\delta} \end{cases} \tag{9}$$

where $\bar{\delta} = \tan\left(\frac{\delta}{2}\right)$ and $a_d^{(n)}$, $b_d^{(n)}$, $a_{k,d}^{(n)}$, $b_{k,d}^{(n)}$, $c_{k,d}^{(n)}$ $(k = 1,\ldots,K)$ are real-valued coefficients depending on $\boldsymbol{x}_{-d}^{(n)}$. Precisely, given an optimal solution $t^\star$ to $\hat{\mathcal{P}}_{d,\boldsymbol{x}^{(n)}}$, $x^\star_{d,n+1} = 2\arctan(t^\star)$ is an optimal solution to $\bar{\mathcal{P}}_{d,\boldsymbol{x}^{(n)}}$. Moreover, if the feasible set of $\hat{\mathcal{P}}_{d,\boldsymbol{x}^{(n)}}$ is unbounded and $v(\hat{\mathcal{P}}_{d,\boldsymbol{x}^{(n)}}) = a_d^{(n)}$, $x^\star_{d,n+1} = \pi$.

Proof. See.[16] □

As shown in[16] the feasible set of $\hat{\mathcal{P}}_{d,\boldsymbol{x}^{(n)}}$ can be expressed as union of disjoint closed intervals, i.e.,

$$\mathcal{F} = \cup_{i=1}^{K_1}\mathcal{B}_i, \tag{10}$$

with $K_1 \leq K+1$ and $\mathcal{B}_i$, $i = 1,\ldots,K_1$, a closed interval whose lower and upper boundaries depend on the specific parameter values of Problem $\hat{\mathcal{P}}_{d,\boldsymbol{x}^{(n)}}$. Since, $\mathcal{F}$ can be evaluated with an overall computational complexity $O(K \log_2(K))$ and the objective function of Problem $\hat{\mathcal{P}}_{d,\boldsymbol{x}^{(n)}}$ exhibits some monotonicities, it is shown in[16] that the optimal solution to $\hat{\mathcal{P}}_{d,\boldsymbol{x}^{(n)}}$ can be obtained in polynomial time. Finally, to initialize **Algorithm 1**, two heuristic procedures are considered. The former relies on the SemiDefinite Relaxation (SDR) and randomization paradigm, where the SDR of $\mathcal{P}$ is obtained removing both the similarity and the rank constraint. The latter considers a radar code design aimed at minimizing the energy transmitted on both licensed and jammed bandwidths, so as to reduce both produced and perceived interference, while fulfilling the constant modulus and similarity constraints. In particular, an iterative procedure based on alternating optimization is devised to handle the resulting optimization problem.

4. PERFORMANCE ANALYSIS

In this section the performance assessment of **Algorithm 1** is conducted. Specifically, achievable detection performance, spectral shape, and autocorrelation features, associated with the devised waveforms are analyzed. To this end, a radar whose baseband equivalent transmitted signal has a two-sided bandwidth of 810 kHz is considered. The disturbance covariance matrix is described as $\boldsymbol{M} = \sigma_0 \boldsymbol{I} + \sum_{k=1}^{K} \frac{\sigma_{I,k}}{\Delta f_k} \boldsymbol{R}_I^k + \sum_{k=1}^{K_J} \sigma_{J,k} \boldsymbol{R}_{J,k}$, where $\sigma_0 = 0$ dB is the thermal noise level; $K = 7$ is the number of licensed emitters; for $K = 1, \ldots, 7$, $\sigma_{I,k}$ accounts for the energy of the k-th coexisting telecommunication network operating on the normalized frequency band $[f_2^k, f_1^k]$ with $\Delta f_k = f_2^k - f_1^k$ the related bandwidth extent; $K_J = 2$ is the number of active and unlicensed narrowband interference sources; $\sigma_{J,k}$, $k = 1, \ldots, K_J$, accounts for the energy of the k-th interference source ($\sigma_{J,1\,\mathrm{dB}} = 40$ dB, $\sigma_{J,2\,\mathrm{dB}} = 50$ dB); $\boldsymbol{R}_{J,k}$, $k = 1, \ldots, K_J$, is the normalized disturbance covariance matrix of the k-th interference source, whose normalized carrier frequency and bandwidth are $f_{J,k}$ and $\Delta_{J,k}$ ($f_{J,1} = 0.7025$, $f_{J,2} = 0.7525$ and $\Delta_{J,k} = 510^{-3}$, $k = 1, 2$).

As to the overlaid telecommunication systems spectrally coexisting with the radar of interest, the following normalized baseband equivalent radar stop-bands are considered: $\Omega_1 = [0.06, 0.11]$, $\Omega_2 = [0.14, 0.26]$, $\Omega_3 = [0.27, 0.29]$, $\Omega_4 = [0.31, 0.37]$, $\Omega_5 = [0.39, 0.46]$, $\Omega_6 = [0.48, 0.51]$, $\Omega_7 = [0.82, 0.87]$. Hence, it is required that the radar probing waveform fulfills the spectral compatibility requirements corresponding to $E_{I\,dB}^k = 10 \log_{10}(E_I^k) = -30$ dB for $k = 3, 6$ and $E_{I\,dB}^k = -20$ dB for the other frequency bands. Finally, concerning the reference code $\boldsymbol{c}_0$, it is employed a unitary norm Linear Frequency Modulated (LFM) pulse with a duration of 200 μs and a chirp rate $K_s = (750 \times 10^3)/(200 \times 10^{-6})$ Hz/s which corresponds to $N = 162$ samples due to the considered sampling frequency.

In Figure 1 the probability of detection (P_d) versus $|\alpha_T|^2$ (assuming a false alarm probability of $P_{fa} = 10^{-4}$) is shown considering different similarity parameter values*. For comparison, it is also plotted the P_d associated with the upper bound to the normalized SINR provided by the SDR of $\mathcal{P}$. The results show that increasing the similarity parameter better and better performance are obtained as a result of the larger degrees of freedom available at the design stage. Interestingly, as ϵ is large enough, i.e. $\epsilon \geq 1.405$ for the case study under test, the upper bound performance is substantially achieved highlighting the effectiveness of the proposed algorithm.

*Note that for any ϵ, **Algorithm 1** is run with three different initializations: the first is obtained via SDR and randomization approach, the second minimizing the energy transmitted on the occupied bandwidths, and third corresponds to the optimized code at the previous ϵ value. Hence, among the three synthesized codes the one providing the higher $\overline{\text{SINR}}$ is picked up.

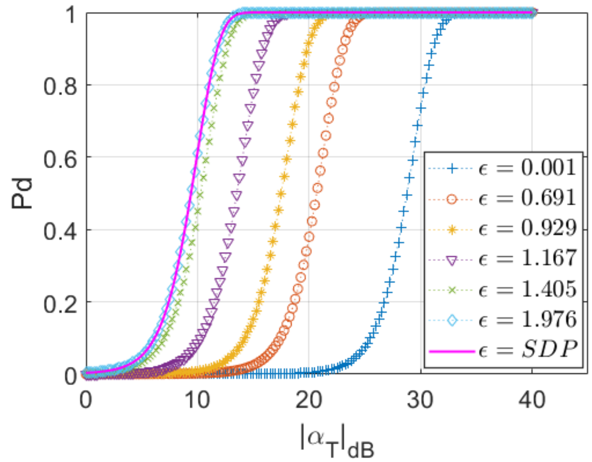
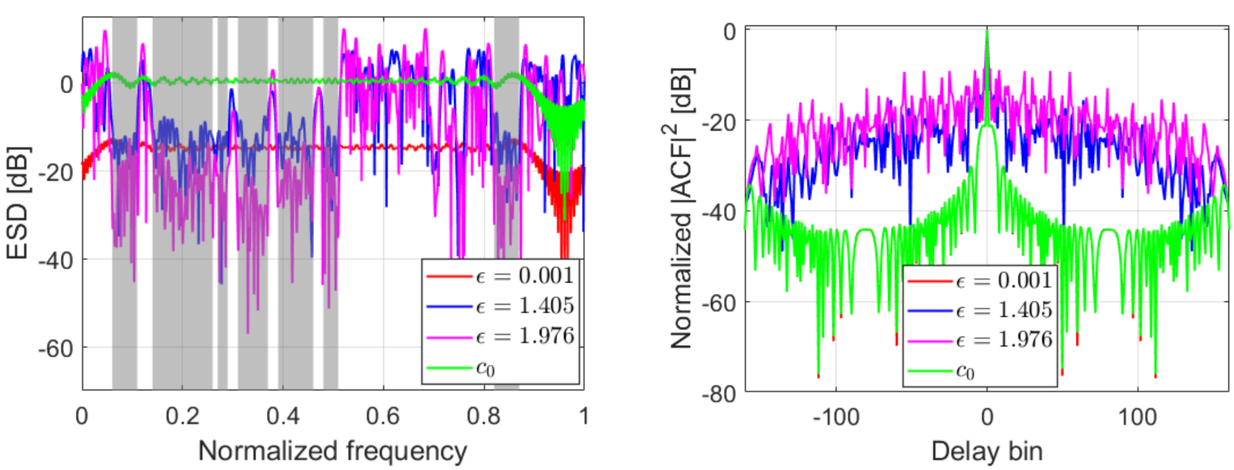


Figure 1. P_d versus $|\alpha_T|^2$ (in dB) of codes synthesized for different ϵ, assuming $P_{fa} = 10^{-4}$.

In Figure 2, the behavior of the codes synthesized through **Algorithm 1** (in terms of ESD and normalized ACF) is reported for three different values of epsilon, i.e., $\epsilon \in \{0.001, 1.405, 1.976\}$. In Figure 2(a), the stop-bands (i.e., where the licensed systems are located) are shaded in light gray. Therein, the curves highlight the capability of the devised technique to control the amount of interference energy produced over the shared frequency bandwidths, as required by the enforced local spectral compatibility constraints. In this respect, note that the code designed for $\epsilon = 0.001$ is almost aligned with $\boldsymbol{c}_0$ and experiences just an energy modulation to satisfy the forced spectral constraints. Moreover, an improvement in the "useful" energy distribution is observed as the similarity parameter increases. As a result, interference rejection of unlicensed sources is achieved via a reduction of the radar emission in correspondence of the shared frequencies. Finally, inspection of Figure 2(b) suggests that better P_d values and subsequently, interference rejection are traded for ideal range resolutions and/or ISL/PSL.



(a) (b)

Figure 2. a) ESD; b) Squared modulus of the ACF. Green curve: reference code $\boldsymbol{c}_0$; red curve: $\epsilon = 0.001$; blue curve: $\epsilon = 1.405$; magenta curve: $\epsilon = 1.976$.

5. CONCLUSION

Radar waveform design intended for addressing the spectrum congestion problem while complying with the current amplifiers technology has been explored. Specifically, multiple spectral compatibility constraints have been enforced at the design stage to ensure a tight and local control on the interference energy induced on each shared/reserved bandwidth. Furthermore, to account for hardware limitations, the radar waveform is constrained to the class of constant modulus signals. Hence, a polynomial computational complexity solution technique, with ensured convergence properties, has been developed to synthesize optimized radar waveforms in terms of achieved SINR. The performance of the devised signals has been analyzed studying the trade-off among the achievable detection probability, spectral shape, and ACF features. As possible future research line, it might be worth to account for the presence of signal-dependent interference at the design stage.

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