Risk Sensitive Shifted Rayleigh Filter for Underwater Bearings-Only Target Tracking Problems

Nikhil Sharma, Ranjeet Kumar Tiwari and Shovan Bhaumik
Indian Institute of Technology Patna, Patna-801106, India

Abstract

Tracking with bearings-only measurement (BOT) is a challenging filtering problem since many years. It is observed that the angle measurements do not always adhere to the assumed statistics during the estimation interval. Sometimes, for a particular interval of time, an un-modeled measurement spike or exploding variance is observed, which corrupts the estimation accuracy severely. In this work, we develop a robust shifted Rayleigh filter (SRF) based on exponential quadratic cost criteria for a BOT problem with such corrupted measurements.

System Model

- The target model is given by

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} - \boldsymbol{\mho}_{k-1,k} + \mathbf{w}_{k-1}, \quad (1)$$

where $\mathbf{x}_k$ is the relative state vector between target and observer and is expressed as $[\mathrm{x_k}, \mathrm{y_k}, \mathrm{v_{x_k}}, \mathrm{v_{y_k}}]^T$ and

$$\mathbf{F} = \begin{bmatrix} I_{2\times2} & \Delta T I_{2\times2} \\ O_{2\times2} & I_{2\times2} \end{bmatrix}.$$

- $\mathbf{w}_{k-1} \sim \mathcal{N}(0, \mathbf{Q})$ and

$$\mathbf{Q} = \begin{bmatrix} \frac{\Delta T^3}{3} I_{2\times2} & \frac{\Delta T^2}{2} I_{2\times2} \\ \frac{\Delta T^2}{2} I_{2\times2} & \Delta T I_{2\times2} \end{bmatrix} \tilde{q}.$$

- $\boldsymbol{\mho}_{k-1,k}$ represents a vector of deterministic inputs from the observer motion.
- The measurement is the angle with respect to true north direction and is given as

$$\theta_k = \tan^{-1}\left[\frac{\mathrm{x_k}}{\mathrm{y_k}}\right] + v_k, \quad (2)$$

where $v_k \sim \mathcal{N}(0, R)$.

Problem

It is observed that the received measurements do not adhere with Eqn. (2) and spike like measurements which lasts for a specific sampling time intervals are received as shown in the Figure 1. Due to this spike, we observed considerable divergence while using the conventional filters for tracking.

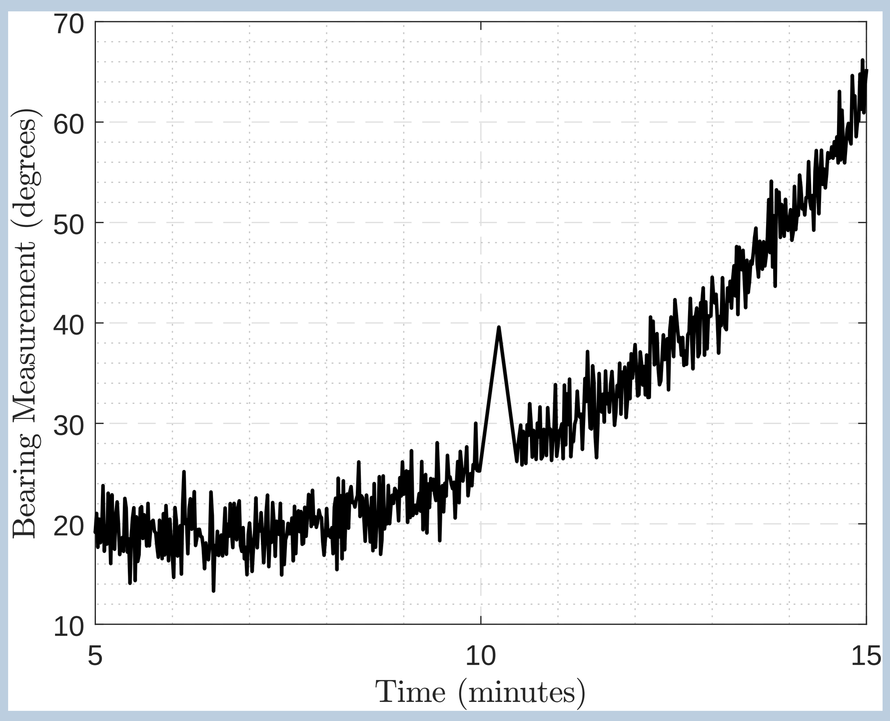

Fig. 1: Measurement Spike

References

[1] Boel, R. K., James, M. R., and Petersen, I. R., "Robustness and risk-sensitive filtering," *IEEE Transactions on Automatic Control* **47**(3), 451–461 (2002).

[2] Bhaumik, S., Sadhu, S., and Ghoshal, T., "Risk-sensitive formulation of unscented Kalman filter," *IET control theory & applications* **3**(4), 375–382 (2009).

[3] Clark, J., Vinter, R., and Yaqoob, M., "Shifted Rayleigh filter: A new algorithm for bearings-only tracking," *IEEE Transactions on Aerospace and Electronic Systems* **43**(4), 1373–1384 (2007).

Solution Approach

- The shifted Rayleigh filter framework is employed which introduces an augmented measurement equation and calculates exact posterior density of estimates given a Gaussian distributed prior density.
- The augmented measurement is given by

$$\boldsymbol{\alpha}_k = [\boldsymbol{I}_{r\times r}\ \boldsymbol{O}_{r\times(n-r)}]\mathbf{x}_k + \boldsymbol{\gamma}_k \quad (3)$$

- Robustness is imparted by using an exponential cost function [1]

$$J_k(\hat{\mathbf{x}}_k) = \mathbb{E}\left[\exp\left\{\mu_1 \sum_{i=0}^{k-1}(\mathbf{x}_i - \hat{\mathbf{x}}_i)^T(\mathbf{x}_i - \hat{\mathbf{x}}_i) + \mu_2(\mathbf{x}_k - \hat{\mathbf{x}}_k)^T(\mathbf{x}_k - \hat{\mathbf{x}}_k)\right\}\right]. \quad (4)$$

- The linear process model (1), augmented measurements and information state are exploited to arrive at the estimator equations for the risk-sensitive shifted Rayleigh filter (RS-SRF).

Information State

- Information state is defined as

$$\sigma_k = \exp\left\{\mu_1 \sum_{i=0}^{k-1}(\mathbf{x}_i - \hat{\mathbf{x}}_i)^T(\mathbf{x}_i - \hat{\mathbf{x}}_i)\right\} p(\mathbf{x}_k|\boldsymbol{\alpha}_k). \quad (5)$$

- It is assumed that σ_k is an unnormalized Gaussina distribution with $\sigma_0 = p(\mathbf{x}_0)$
- Since we have used the conditional pdf as the information state, it follows the recursion

$$\sigma_{k+1} = \int p(\boldsymbol{\alpha}_{k+1}|\boldsymbol{\alpha}_k, \mathbf{x}_{k+1})p(\mathbf{x}_{k+1}|\mathbf{x}_k)\sigma_k d\mathbf{x}_k. \quad (6)$$

- $p(\boldsymbol{\alpha}_{k+1}|\boldsymbol{\alpha}_k, \mathbf{x}_{k+1})$ is constructed using (3). γ_k is the noise associated with augmented measurement model which is assumed as $\gamma_k \sim \mathcal{N}(0, \mathbf{R_a})$.

Modified Cost Function

- From (4) and (5), the cost function can now be re-written as

$$J_k(\hat{\mathbf{x}}_k) = \int_{\mathbb{R}^n} \exp\left(\mu_2(\mathbf{x}_k - \hat{\mathbf{x}}_k)^T(\mathbf{x}_k - \hat{\mathbf{x}}_k)\right)\sigma_k d\mathbf{x}_k.$$

- Subsequently, the minimum cost estimate is expressed as

$$\hat{\mathbf{x}}_k = \arg\min_{r\epsilon\mathbb{R}^n} \int \exp(\mu_2(\mathbf{x}_k - r)^T(\mathbf{x}_k - r))\sigma_k d\mathbf{x}_k.$$

Algorithm

- `Initialize with` $\hat{\mathbf{x}}_{0|0}$ `and covariance` $\boldsymbol{\Sigma}_{0|0}$
- *for* $k = 1, \cdots, k_{max}$

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}\hat{\mathbf{x}}_{k-1|k-1} - \boldsymbol{\mho}_{k-1,k}$$
$$\boldsymbol{\Sigma}_{k|k-1} = \mathbf{F}[(\boldsymbol{\Sigma}^{-1}_{k-1|k-1} - 2\mu_1 I)^{-1}]\mathbf{F}^T + \mathbf{Q}$$
$$\mathbf{K}_k = \boldsymbol{\Sigma}_{k|k-1}\mathbf{H}^T\mathbf{V}_k^{-1}$$
$$\hat{\mathbf{x}}_{k|k} = (I - \mathbf{K}_k\mathbf{H})\hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k E[r_k|\boldsymbol{\beta}_k]\mathbf{V}_k^{1/2}\boldsymbol{\beta}_k$$
$$\boldsymbol{\Sigma}_{k|k} = (I - \mathbf{K}_k\mathbf{H})\boldsymbol{\Sigma}_{k|k-1} + cov(r_k|\boldsymbol{\beta}_k)\mathbf{K}_k\mathbf{V}_k^{-1/2}\boldsymbol{\beta}_k{\boldsymbol{\beta}_k}^T\mathbf{V}_k^{1/2}\mathbf{K}_k^T,$$

`where` $\mathbf{V}_k = \mathbf{H}\boldsymbol{\Sigma}_{k|k-1}\mathbf{H}^T + R$, $\boldsymbol{\beta}_k$ `is projection of` $\boldsymbol{\alpha}_k$ `on the unit circle, and` $\mathbb{E}[r_k|\boldsymbol{\beta}_k]$ `and` $cov[r_k|\boldsymbol{\beta}_k]$ `are computed as given in` [3].

- *end*

Simulation Results

- Fig. 2 represents the BOT scenario. The measurement spike is included between $t_s =$ 600th and $t_e =$ 660th steps in Fig. 1

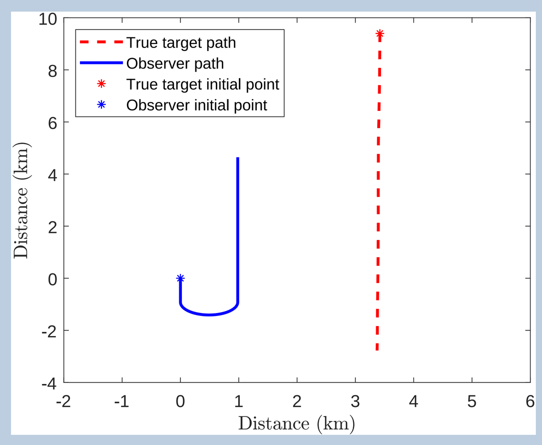

Fig. 2: Tracking scenario

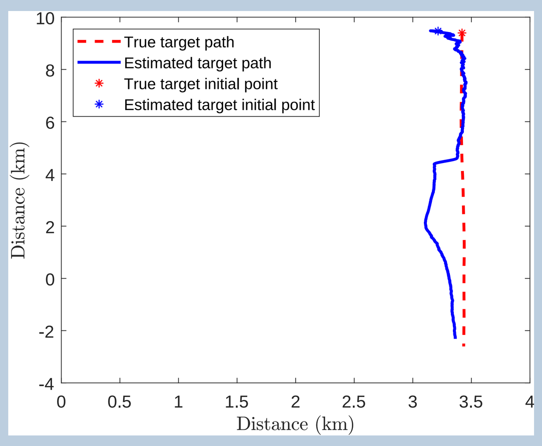

Fig. 3: Estimated target path

- The risk sensitive unscented Kalman filter (RS-UKF) [2] and the shifted Rayleigh filter (SRF) have been chosen for comparison in terms of the root mean square error (RMSE) and the track loss. The risk sensitive parameter, $\mu_1 = -5$, is used in this simulation.

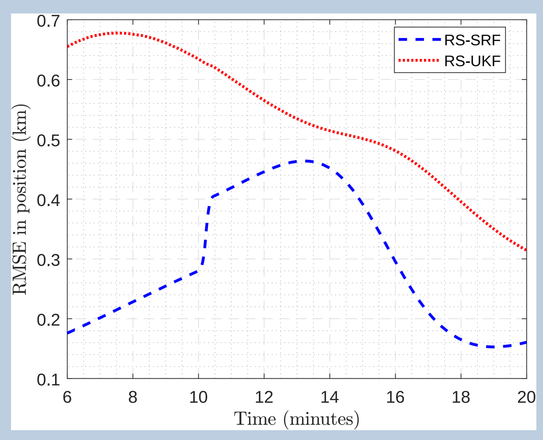

Fig. 4: RMSE position

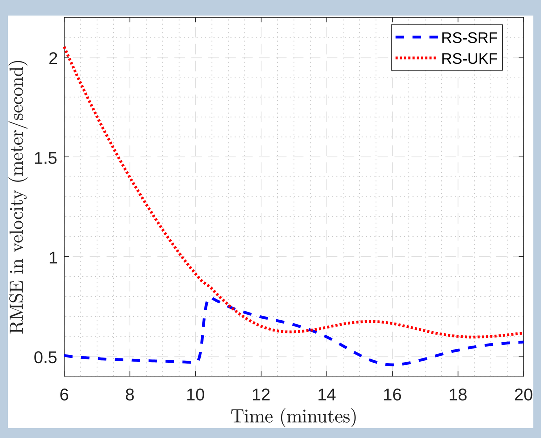

Fig. 5: RMSE velocity

- Table 1 presents the terminal RMSE in position and velocity along with the track losses for different filtering algorithms.

Table 1: Terminal RMSE in position and velocity, and % track-loss for different filters

	$\mathrm{RMSE_{pos}}$	$\mathrm{RMSE_{vel}}$	Track loss
RS-SRF	292	1.28	0.0%
RS-UKF	308	1.31	6.7%
SRF	–	–	100%

Conclusions

1. Conventional moment matching filters have shown high track divergence while dealing with the measurement spike.
2. An exponential quadratic cost function is employed to impart robustness.
3. The risk sensitive parameter appears in estimator equations can tackle the measurement uncertainties and can be tuned for better results.
4. The simulation results suggests the use of the RS-SRF if presence of the measurement spike is likely.