# 1st Maritime Situational Awareness Workshop MSAW 2019

# A Constrained Least Squares Approach for 2D PBR Localization

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- Introduction
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# Introduction

A new algorithm for **Passive Bistatic Radar (PBR)** localization exploiting multiple illuminators of opportunity is proposed.

To capitalize **a-priori information** on the receiving **antenna beampattern angular extent**, specific constraints are forced to the target localization process.

The elliptic positioning problem is formulated according to the **constrained Least Squares (LS)** framework.

**MAIN ACHIEVEMENT**

The resulting non-convex optimization problem is globally solved providing a closed-form estimate to the target Cartesian coordinates.

The performance of the new estimator is assessed in terms of **Root Mean Square Error (RMSE)** behavior.

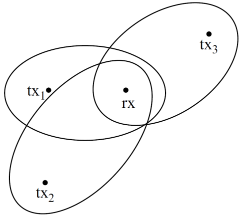
# PBR Localization

PBR-based localization is accomplished using **bistatic target range measurements** from **one/multiple illuminators of opportunity**.

PBR receivers are equipped with **two receiving channels** per transmitter of oppurtunity: one is used to acquire the direct path signal from the selected emitter, the other to gather the induced echoes.

Each bistatic pair of measurements allows to localize the target over an **ellipsoid** (ellipse in a 2D geometry).

The **intersection** of multiple ellipsoids due to different illuminator/passive receiver pairs paves the way for **target positioning**.

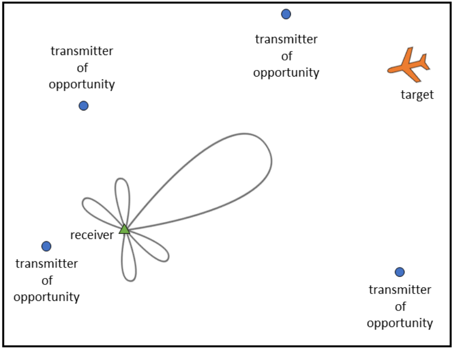



Localization using three transmit-receive pairs.

# System Model

Consider a 2D PBR exploiting multiple transmitters of opportunity.

- $(x_p, y_p) \in \mathbb{R}^2$ the target position, and $\boldsymbol{p} = [x_p, y_p]^T$ the target position vector;
- $(x_0, y_0) = (0, 0)$ the receiver position, coinciding with the reference system origin;
- $(x_{t_i}, y_{t_i}) \in \mathbb{R}^2$ the position of the $i$-th transmitter of opportunity, $i = 1, \ldots, N$;
- $L_i = \sqrt{(x_{t_i} - x_0)^2 + (y_{t_i} - y_0)^2}$ the distance between the $i$-th transmitter and the receiver.



# Observation Model

At the receiver side, after the classic PBR **cross-correlation** based processing, the following $N$ **delay/range measurements** are available

$$\tau_i = \tilde{\tau}_i + n_i, \;\; i = 1, \dots, N$$

where

$$\tilde{\tau}_i = \frac{1}{c}\left(\|\boldsymbol{p}\| + \sqrt{(x_p - x_{t_i})^2 + (y_p - y_{t_i})^2} - L_i\right)$$

with $c$ the speed of light and $n_1, \dots, n_N$, **statistically independent** zero-mean random variables with variance $\sigma_1^2, \dots, \sigma_N^2$. In particular,

$$\sigma_i = \frac{\sqrt{2}}{B_i\sqrt{\mathrm{SNR}_i}}, \;\; i = 1, \dots, N$$

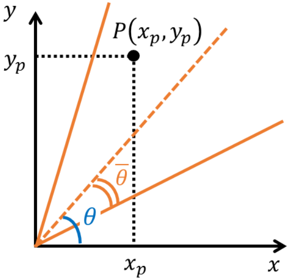
where $B_i$ is the **frequency bandwidth** of the $i$-th signal of opportunity and $\mathrm{SNR}_i$ is the **Signal to Noise Ratio (SNR)** of the $i$-th bistatic pair evaluated according to the bistatic radar range equation.

# Receiver Beampattern Constraint

Some constraints able to capitalize a-priori information on antenna beampattern angular extent are formalized with the goal of improving target localization reliability.

Let us denote by:

- $\bar{\theta}$ the receiving (half-side) antenna beamwidth, with $0 \leq \bar{\theta} < \pi/2$;
- $\theta \in ]-\pi, \pi[$ the squint angle of the antenna boresight with respect to the $x$-axis;
- For $(x, y) \neq (0, 0)$, $\theta_p$ the target angle of arrival.




We suppose that the target is within the receiving antenna main-beam

$$\theta - \bar{\theta} \leq \theta_p \leq \theta + \bar{\theta}$$

The detector performs correctly its task, i.e., the target triggering the detection resides in the antenna main-beam

# Problem Formulation

Target localization can be formulated as the following constrained LS Problem

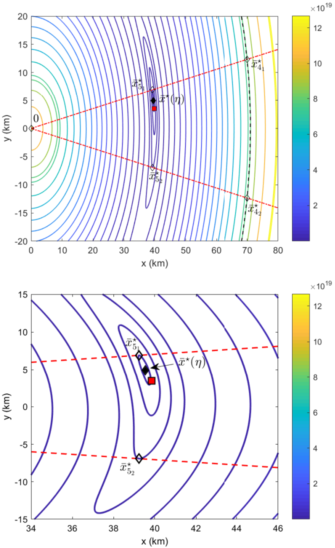
$$\mathcal{P}\begin{cases} \min\limits_{\tilde{\boldsymbol{p}}} & \left\|\tilde{\boldsymbol{A}}\tilde{\boldsymbol{p}} - \boldsymbol{g}\right\|^2 \\ \text{s.t.} & \tilde{\boldsymbol{p}}^T \boldsymbol{B}\tilde{\boldsymbol{p}} = 0 \\ & 0 \leq \tilde{p}_3 \leq b \\ & -\tilde{p}_1\gamma \leq \tilde{p}_2 \leq \tilde{p}_1\gamma \\ & \tilde{p}_1 \geq 0 \end{cases}$$

where $\tilde{\boldsymbol{A}}$ is a model matrix, $\boldsymbol{B}$ is a constraint matrix, $\boldsymbol{g}$ and $b$ are functions of measurements and system parameters, $\gamma$ is related to the receiving beampattern mainlobe size, $\tilde{\boldsymbol{p}}$ is the optimization variable.

- $\tilde{\boldsymbol{p}}$ is a 3D vector with the first two components equal to the target coordinates while the third is an auxiliary variable.

Problem $\mathcal{P}$ is a non-convex optimization problem apparently difficult to solve.

# Solution Technique



## Proposition

An optimal solution to $\mathcal{P}$ belongs a finite set of feasible points (whose cardinality is at most thirteen) which can be computed in closed form.

The corresponding **global optimum** search procedure is referred to as **Angular Constrained Least Squares (ACLS)** method.

The figure show **isolines** of the objective function (entire considered scenario and a zoomed part):

- $\Diamond$-marker: **possible solutions of ACLS**;
- $\blacklozenge$-marker: **selected solution of ACLS**;
- $\square$-marker: **true target position**.

## Solution Technique

**Remark 1**

ACLS method does not require any knowledge about the measurement accuracies.

**Remark 2**

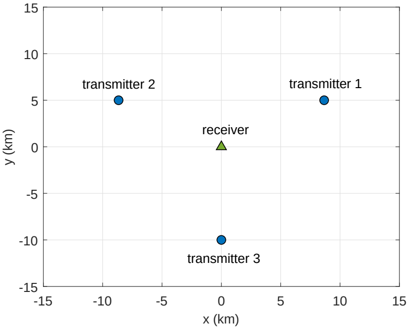
ACLS method provides the sought position estimate in closed-form via computations involving only elementary functions.

# Performance Analysis

A **localization scenario** accounting for $N = 3$ omni-directional broadcast transmitters of opportunity is analyzed.

The transmitters are located at the **vertices** of an **equilateral triangle** whose barycenter is the position of the receiver, that coincides with the origin of the reference system.

The **distance** of each transmitter of opportunity from the receiver is $L_i = 10$ km, $i = 1, 2, 3$.



# Performance Analysis

The performance is assessed considering as figure of merit the **RMSE of the target position estimate**.

Due to the lack of a closed-form expression for the RMSE, **Monte Carlo simulation** method is employed, performing 1000 independent runs.

The analyses are conducted in comparison with the **unconstrained LS**, **TSE (Two-Step Estimation)** algorithms and the **CRLB** benchmark, i.e.

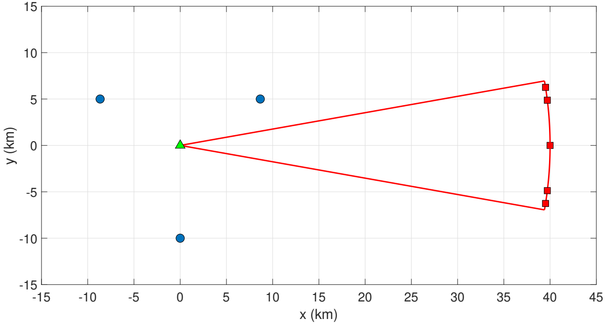
$$\boxed{\sqrt{tr\left(\mathrm{FIM}^{-1}\right)}}$$

with FIM the **Fisher Information Matrix**.

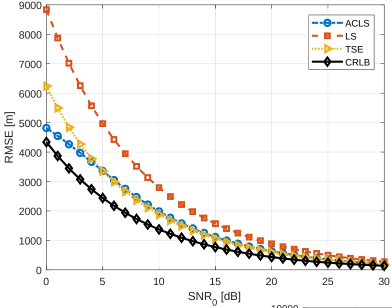
# Results

The following study considers:

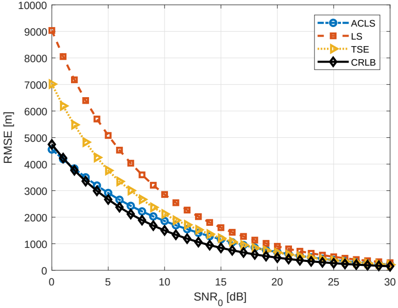
- a **target** located at $(x_p, y_p) = (r\cos\theta_p, r\sin\theta_p)$ with $r = 40$ km and different values of $\theta_p$ ($\theta_p = 0, 7, 9$ deg);
- a **receiving antenna** steered at $\theta = 0$ deg with a main-beam width of $\bar{\theta} = 10$ deg;
- $B_i = 150$ kHz, $i = 1, 2, 3$ (representative of **FM radio stations**).



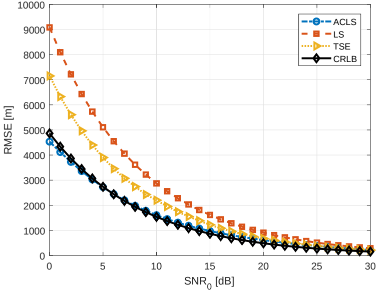
# Results




$\theta_p = 0^\circ$




$\theta_p = 7^\circ$




$\theta_p = 9^\circ$

# Conclusion

- An **innovative** approach for **elliptic location** has been proposed with reference to a PBR receiver that exploits multiple transmitters of opportunity.
- Constraints have been enforced to the target localization process in order to **exploit a-priori information** on the receiving antenna main-beam size.
- The problem has been formulated as a **constrained LS estimation** whose optimal solution provides the Cartesian coordinates of the target.

**MAIN TECHNICAL ACHIEVEMENT**

It has been shown that the position estimate belongs to a finite set (with cardinality at most thirteen) of feasible points obtainable in closed-form via elementary functions.

The conducted analyses have demonstrated the **effectiveness** of the proposed algorithm also in comparison with other **counterparts** available in the open literature especially for low SNRs.

**THANK YOU FOR THE KIND ATTENTION**