Multi-Waveform Active Sonar Tracking

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June 2007

Originally presented at:

Third International Waveform Diversity and Design Conference, Pisa, Italy,
4-8 June 2007
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Abstract—This paper extends the NURC distributed multi-hypothesis tracking technology to include Doppler sensitive (CW) processing. The assessment of the value-added of CW processing in automated undersea detection and tracking is ongoing.

I. INTRODUCTION

In recent years, the NATO Undersea Research Centre (NURC) has developed a multi-hypothesis tracker for multistatic active sonar surveillance networks [1]. Key features of the algorithm include: (1) measurement covariance error expressions that characterize contact localization uncertainty with statistical consistency accounting for numerous system and environmental errors; (2) EKF-based recursive filtering; (3) efficient multi-dimensional data correlation with a linear programming based relaxation approach; (4) flexible, modular fusion architecture for distributed processing.

A tracking example is illustrated in figure 1. Simulated multistatic contacts based on three platforms (9 source-receiver pairs) are shown in magenta; platform ground truth trajectories are in red, and tracks on two mobile and one fixed target are in blue. Note the dramatic false-object reduction and small localization error of tracks as compared with the localization error of target contacts. This tracker has been applied in real-time sea-trial experiments with both towed-array and deployable surveillance equipment, demonstrating the benefit of automated fusion and tracking on onboard signal and information processing [2].

We have developed a simple analytical model for tracker performance as a function of scenario characteristics and key tracker parameters [3]. This model is qualitatively consistent with our experimental results. In particular, we find that centralized tracking outperforms distributed tracking in high FAR situations, while distributed tracking outperforms centralized tracking when target-fading effects dominate. The model can be used to support architectural and parametric choices prior to or during sea-trial operations.

To date, our analysis of distributed fusion architectures has employed at most one tracker for contact data from each source-receiver pair in the surveillance network, followed by scan-based (real-time) track fusion. In this paper, we document two extensions, both of which explore more complex processing architectures: multi-band FM processing, and CW processing that entails Doppler-aided tracking.

The paper is organized as follows. Section 2 discusses briefly our work in multi-band FM processing, and provides references to more detailed discussions. Sections 3-4 provide algorithmic details for CW-aided tracking, and section 5 provides a preliminary performance assessment and future directions.

II. MULTI-BAND FM TRACKING

The first extension leverages sub-band processing research at NURC that seeks to identify optimal transmitted pulse bandwidths. In particular, for a given source-receiver pair, we compare broadband FM detection and tracking with a number of alternative architectures, all of which are based on contact data from a number of sub-bands. The towed-array dataset that we consider has twenty sub-bands derived from the broadband signal. We consider the following processing architectures:

- Single-sub-band tracking: contact files for each source/receiver/sub-band triple are processed by separate tracking modules;
- **Centralized tracking**: the full sequence of contact files from all source/receiver/sub-band triples is processed by a single tracking module;
- **Track fusion**: the tracks generated by the twenty single-sub-band tracking modules are processed by a second-stage tracking module;
- **Grid-based static fusion**: for each broadband ping, the twenty sub-band contact files are combined with a simple M-of-N fusion rule applied over a grid in time-bearing space, and the fused contact files are processed by a second-stage tracking module;
- **Static fusion**: as a variation on the previous case, we apply the tracker with a two-pass centralized methodology that corresponds to (grid-less) static fusion followed by tracking over time.

An illustration of these processing schemes is given in figure 2. While our initial results indicate that broadband detection and tracking continues to outperform the more elaborate architectures, the results suggest that further work to determine optimal bandwidths may provide a competitive alternative, particularly if a range of optimal center frequencies can be identified. Details on processing and performance results are documented in [4-5]. The issue of performance limitations when processing high-rate contact files through the tracker is addressed in [6].

![Fig. 2. Schematic representation of a number of approaches to fusion and tracking of FM contacts.](image)

**TABLE I. SENSOR MEASUREMENT INFORMATION.**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Measured value [units]</th>
<th>Error covariance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ping time</td>
<td>$t$ [sec]</td>
<td>none</td>
</tr>
<tr>
<td>Source location</td>
<td>$x^S, y^S$ [m]</td>
<td>$\sigma^2_{x^S}, \sigma^2_{y^S}$</td>
</tr>
<tr>
<td>Source velocity</td>
<td>$\dot{x}^S, \dot{y}^S$ [m/s]</td>
<td>not used</td>
</tr>
<tr>
<td>Receiver location</td>
<td>$x^R, y^R$ [m]</td>
<td>$\sigma^2_{x^R}, \sigma^2_{y^R}$</td>
</tr>
<tr>
<td>Receiver velocity</td>
<td>$\dot{x}^R, \dot{y}^R$ [m/s]</td>
<td>not used</td>
</tr>
<tr>
<td>Array orientation</td>
<td>$\phi^R$ [rad]</td>
<td>$\sigma^2_{\phi^R}$</td>
</tr>
<tr>
<td>Sound speed</td>
<td>$c$ [m/s]</td>
<td>$\sigma^2_c$</td>
</tr>
<tr>
<td>Timing</td>
<td>$\tau$</td>
<td>$\sigma^2_{\tau}$</td>
</tr>
<tr>
<td>Relative bearing</td>
<td>$\theta$</td>
<td>$\sigma^2_{\theta}$</td>
</tr>
<tr>
<td>Doppler (shift relative to transmitted CW frequency)</td>
<td>$\frac{\Delta f}{f_0}$</td>
<td>$\sigma^2_{\Delta f/f_0}$</td>
</tr>
</tbody>
</table>

The derived measurements that are convenient for use in nonlinear filtering are the following:

$$r = \frac{a}{b},$$  \hspace{1cm} (1)

$$\eta = \theta + \phi,$$  \hspace{1cm} (2)

$$\tilde{\eta} = -\frac{\Delta f}{2f_0} c,$$  \hspace{1cm} (3)

where

$$a = c^2 \tau^2 - \Delta^2,$$  \hspace{1cm} (4)

$$b = 2(c \tau - \Delta \cos \alpha),$$  \hspace{1cm} (5)

$$\alpha = \tan^{-1}\left(\frac{y^S - y^R}{x^S - x^R}\right) - \theta - \phi^R,$$  \hspace{1cm} (6)

$$\Delta = \left((x^S - x^R)^2 + (y^S - y^R)^2\right)^{1/2}.$$  \hspace{1cm} (7)

III. CW MEASUREMENT MODEL

The second extension that we consider leverages Doppler-sensitive processing based on CW transmissions [7]. It is well known that FM and CW waveforms have complementary characteristics in terms of detection performance. As with sub-band FM processing, the use of simultaneous FM and CW transmissions allows for more general fusion architectures than previously considered. CW processing requires an extension to our statistically consistent contact measurement model to include bistatic Doppler in addition to timing and bearing, and a corresponding augmentation of our EKF filter.

Each contact includes the sensor measurements listed in table 1, along with the corresponding error covariances.
Equation (3) provides the derived bistatic range rate measurement. The measurement covariance matrix is given by

\[
R_{ij} = \begin{bmatrix}
\sigma_i^2 & \sigma_{ir} & \sigma_{i\theta} \\
\sigma_{ir} & \sigma_{rr} & \sigma_{r\theta} \\
\sigma_{i\theta} & \sigma_{r\theta} & \sigma_{\theta\theta}
\end{bmatrix},
\]

(7)

where

\[
\sigma_i^2 = \frac{b^2\sigma_a^2 + a^2\sigma_b^2 - 2ab\sigma_{ab}}{b^2},
\]

(8)

\[
\sigma_{rr} = \sigma_{\theta\theta} + \sigma_{\phi\phi},
\]

(9)

\[
\sigma_{ir} = \frac{\Delta f^2}{f_0 b^2} (\alpha - \tau^2 bc),
\]

(10)

\[
\sigma_{i\theta} = \sigma_{\theta\theta}^2 + \sigma_{\phi\phi}^2,
\]

(11)

\[
\sigma_{r\theta} = 0,
\]

(12)

\[
\sigma_{\theta\theta} = \frac{2}{f_0^2} \sigma_{\phi\phi}^2 + \frac{c^2}{4} \sigma_{\phi\phi}^2.
\]

(13)

In turn, these expression require the following quantities:

\[
\sigma_a^2 = 4\tau^4 c^2 \sigma_c^2 + 4\tau^2 \sigma_c^2 + 4\Delta^2 \sigma_a^2,
\]

(14)

\[
\sigma_b^2 = 4\left(\tau^2 \sigma_c^2 + c^2 \sigma_c^2 + \cos^2 \alpha \sigma_{\alpha}^2 + \Delta^2 \sin^2 \alpha \sigma_{\alpha}^2 - 2 \cos \alpha \sin \alpha \Delta \sigma_{\alpha \alpha}\right),
\]

(15)

\[
\sigma_{ab} = 4 \left(\tau^3 c \sigma_c^2 + \tau^2 c \sigma_c^2 + \Delta \cos \alpha \sigma_{\alpha}^2 \right),
\]

(16)

\[
\sigma_{\alpha}^2 = \frac{1}{\Delta^2} \left[ \left(\sigma_{x,s}^2 + \sigma_{y,s}^2 + \sigma_{x,v}^2 + \sigma_{y,v}^2\right) - 2 \sigma_{x,s} \sigma_{y,s} + 2 \sigma_{x,v} \sigma_{y,v} \right],
\]

(17)

\[
\sigma_{\phi}^2 = \frac{1}{\Delta} \left(\sigma_{x,s}^2 + \sigma_{y,s}^2 + \sigma_{x,v}^2 + \sigma_{y,v}^2\right),
\]

(18)

\[
\sigma_{\phi}^2 = \frac{1}{\Delta^2} \left[ \left(\sigma_{x,s}^2 + 
\sigma_{x,v}^2\right) + \left(\sigma_{y,s}^2 + \sigma_{y,v}^2\right) + 2 \sigma_{x,s} \sigma_{y,s} + 2 \sigma_{x,v} \sigma_{y,v}\right],
\]

(19)

\[
\sigma_{\theta}^2 = \frac{2a}{b^2} \Delta \sin \alpha \sigma_{\phi}^2,
\]

(20)

\[
\sigma_{\phi}^2 = \frac{2a}{b^2} \Delta \sin \alpha \sigma_{\phi}^2.
\]

(21)

When the measured source and receiver locations are the same, we replace equations (14-16) and (20-21) by the following:

\[
\sigma_a^2 = 4\tau^4 c^2 \sigma_c^2 + 4\tau^2 \sigma_c^2 + 4\Delta^2 \sigma_a^2,
\]

(34)

\[
\sigma_b^2 = 4\left(\tau^2 \sigma_c^2 + c^2 \sigma_c^2 + \cos^2 \alpha \sigma_{\alpha}^2 + \Delta^2 \sin^2 \alpha \sigma_{\alpha}^2 - 2 \cos \alpha \sin \alpha \Delta \sigma_{\alpha \alpha}\right),
\]

(35)

\[
\sigma_{ab} = 4 \left(\tau^3 c \sigma_c^2 + \tau^2 c \sigma_c^2 + \Delta \cos \alpha \sigma_{\alpha}^2 \right),
\]

(36)

\[
\sigma_{x,s} = -\frac{2a}{b^2} \left(\sigma_{x,s}^2 \cos(\theta + \phi^k) + \sigma_{x,v} \sin(\theta + \phi^k)\right),
\]

(37)

\[
\sigma_{x,v} = -\frac{2a}{b^2} \left(\sigma_{x,s}^2 \sin(\theta + \phi) + \sigma_{x,v} \cos(\theta + \phi)\right).
\]

(38)

Note the following:

- Array orientation is measured counter-clockwise from the positive x-axis. Relative bearing is measured counter-clockwise from the array orientation.
- We assume all errors are zero mean and uncorrelated. Furthermore, the derivations are valid under small-error assumptions, and are a direct extension to [8], where no Doppler measurements were available.
- Contact relative bearing measurements are based on beamforming that requires use of a local sound speed estimate. The sound speed used below for ranging purposes is a global sound speed estimate; thus we can reasonably assume uncorrelated sound speed and bearing errors.
- The measurement covariance matrix expressions are different in the quasi-monostatic (identical measured source and receiver locations) and bistatic cases. Note that quasi-monostatic is different from a true monostatic system, where source and receiver positional errors are correlated. For near-monostatic cases, it is best to use the quasi-monostatic expressions to avoid numerical instabilities.
- We assume precise knowledge of ping time; errors in corrections for source latency are included as timing errors. (Thus there is a small correlation in timing error from one contact to the next.).
- Interestingly, the derived measurement error covariance expressions do not require knowledge of source and receiver velocity errors. This has to do with our problem formulation. A related issue is to examine the merits of using derived range-bearing measurements rather than time-bearing measurements directly in nonlinear filtering.
- If we wish to account properly for intra-ping effects, the receiver location, velocity, and orientation information must be contact-specific. Regardless, in
our nonlinear filtering, we will continue to assume that targets are ensonified at ping time. This introduces a small error but neglects difficult out-of-sequence measurement issues [9].

- In general, errors in range and bearing are correlated, except in the quasi-monostatic case (identical source and receiver measured locations).
- In the absence of speed of sound error, there is no correlation for range and bistatic Doppler errors.

IV. CW FILTERING AND TRACKING

Neglecting for simplicity the errors in source and receiver locations and in sound speed, we proceed with a Cartesian measurement model, with the following positional estimates and state estimation covariance matrix.

\[ x^T = x^R + r \cos \eta, \]
\[ y^T = y^R + r \sin \eta. \]

The measurement covariance matrix is given by

\[ R_{xy} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & 0 \\ \sigma_{xy} & \sigma_y^2 & 0 \\ 0 & 0 & \sigma_z^2 \end{bmatrix}, \]

where

\[ \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix} = \Sigma \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix} \Sigma', \]

\[ \Sigma = \begin{bmatrix} \cos \eta & -r \sin \eta \\ \sin \eta & r \cos \eta \end{bmatrix}. \]

The EKF design relies on a standard nearly constant velocity target kinematic model, and the parameters listed in Table 2. We process measurements at times \( t_k \), with measurement covariance \( R_k = \begin{bmatrix} R_{xx}^{11} & R_{xx}^{12} \\ R_{xx}^{21} & R_{xx}^{22} \end{bmatrix} \), with sub-matrices to denote the positional and range-rate portions of the matrix. For each time \( t_k \), the filter determines a state estimate \( \hat{X}(k|k) \) and state estimation covariance matrix \( P(k|k) \).

**TABLE II. PARAMETERS IN EXTENDED KALMAN FILTER (EKF).**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Notation [units]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior velocity uncertainty</td>
<td>( \Sigma_v ) (2-by-2) [m²]</td>
</tr>
<tr>
<td>Process noise intensity</td>
<td>( q_x, q_y ) [m²s⁻³]</td>
</tr>
</tbody>
</table>

A. Filter Initialization

\[ X(1|1) = \begin{bmatrix} x_1^T \\ y_1^T \\ 0 \end{bmatrix}, \]

\[ P(1|1) = \begin{bmatrix} R_x^{11} & 0 \\ 0 & \Sigma_r \end{bmatrix}. \]

B. Filter Prediction

\[ X(k+1|k) = \Phi_k X(k | k), \]

\[ P(k+1|k) = \Phi_k P(k | k) \Phi_k' + Q_k, \]

\[ \Phi_k = \begin{bmatrix} 1 & 0 & \Delta t_k & 0 \\ 0 & 1 & 0 & \Delta t_k \\ 0 & 0 & 1 & 0 \end{bmatrix}, \]

\[ Q_k = \begin{bmatrix} \frac{1}{2} q_x (\Delta t_k)^3 & 0 & \frac{1}{2} q_y (\Delta t_k)^2 & 0 \\ 0 & \frac{1}{2} q_x (\Delta t_k)^3 & 0 & \frac{1}{2} q_y (\Delta t_k)^2 \\ \frac{1}{2} q_x (\Delta t_k)^2 & 0 & q_x \Delta t_k & 0 \\ 0 & \frac{1}{2} q_y (\Delta t_k)^2 & 0 & q_y \Delta t_k \end{bmatrix}. \]

C. Filter Update

\[ \begin{bmatrix} x_{k+1}^T \\ y_{k+1}^T \end{bmatrix} = \begin{bmatrix} x_k^T + \frac{1}{2} q_x (\Delta t_k)^3 \\ y_k^T + \frac{1}{2} q_y (\Delta t_k)^2 \end{bmatrix} \]

\[ L(k+1) = \begin{bmatrix} x_{k+1}^T - x(k+1 | k) \\ y_{k+1}^T - y(k+1 | k) \end{bmatrix}, \]

\[ P(k+1|k) = (I - L(k+1)C(k+1)) P(k+1 | k), \]

\[ L(k+1) = P(k+1 | k) \bar{C} \]

\[ C(k+1) = P(k+1 | k) \bar{C} \bar{C}^{-1}, \]

\[ h(X, X^s, X^b) = 2r_T S \begin{bmatrix} 2r_T \dot{x} + \dot{x}^s \\ 2r_T \dot{y} + \dot{y}^s \end{bmatrix} \]

\[ + \begin{bmatrix} \dot{x} + \dot{x}^s \\ \dot{y} + \dot{y}^s \end{bmatrix} \]

\[ C(k+1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \]

\[ C^21(k) C^33(k) C^34(k) \]
D. Data Association

While the derivation of the measurement model and nonlinear filter given above is lengthy, from a computational perspective the challenge in multi-sensor multi-waveform tracking is efficiently and robustly to associate contact-level data, while effectively removing false contacts. The data association methodology detailed in [1-2] is directly applicable; there is slight modification in the track scores, which rely on modified filter residual calculations that reflect bistatic rage rate measurement information.

V. PRELIMINARY PERFORMANCE ASSESSMENT AND RECOMMENDATIONS

Statistical results of simulation-based and sea trial-based evaluation of FM-CW multistatic tracking will be reported shortly [10]. A sample output with simulated FM and CW data, and a centralized MHT architecture is in figure 3. Our input and output performance metrics follow ongoing multilaboratory benchmarking efforts [11]. Our analysis is based on the use of numerous source-receiver-waveform combinations, and leads to these conclusions:

- As in past FM-only tracking, automated fusion and tracking provides a dramatic improvement to detection-level surveillance, with a two orders of magnitude reduction in false objects;
- The use of multiple source-receiver-waveform triples provides improved track PD, with a modest increase in false objects; further, the overall fusion gain (ratio of input to output objects) improves;
- The overall effectiveness of FM and CW detection (from an ROC curve perspective) is comparable.

Performance results are scenario-dependent. In particular, it is of significant interest to continue our analysis with aspect-dependent target strength data (as with a real submarine target), where we expect the complementariness of FM/CW data to provide benefit. Future work will include the development of sensor management algorithms for ping sequencing and waveform selection.

\[
C^{11}(X,X^s,X^r) = \frac{(y-y^s)(x+x^s)-(x-x^s)(y+y^s)}{2r_{TS}^2} + \frac{(y-y^s)(x+x^s)-(x-x^s)(y+y^s)}{2r_{TR}^2},
\]

\[
C^{12}(X,X^s,X^r) = \frac{(x-x^s)(y+y^s)-(x-x^s)(y+y^s)}{2r_{TS}^2} + \frac{(x-x^s)(y+y^s)-(x-x^s)(y+y^s)}{2r_{TR}^2},
\]

\[
C^{33}(X,X^s,X^r) = \frac{(x-x^s)}{2r_{TS}} + \frac{(x-x^r)}{2r_{TR}},
\]

\[
C^{34}(X,X^s,X^r) = \frac{(y-y^s)}{2r_{TS}} + \frac{(y-y^r)}{2r_{TR}},
\]

\[
r_{TS} = \sqrt{(x-x^s)^2 + (y-y^s)^2}, \quad r_{TR} = \sqrt{(x-x^r)^2 + (y-y^r)^2}.
\]

REFERENCES

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**Title**

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**Abstract**

.

**Keywords**


**Issuing Organization**

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